

# **Advanced Dynamic Signal Analysis**

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(Part of CoCo-80 User's Manual)

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# 1. INTRODUCTION

CoCo is a hardware platform that can run in either DSA (Dynamic Signal Analyzer) or VDC (Vibration Data Collector) mode.

This CoCo Advanced DSA Users Manual discusses the theory, EDM software and CoCo operation for the optional advanced dynamic signal analysis features including:

- Swept Sine Analysis
- Acoustic Data Acquisition: Octave Analysis and Sound Level Meter
- Order Tracking
- Sock Response Spectrum Analysis
- Automated Test and Limit Test
- Real Time Digital Filters
- Histogram and Statistics Measures
- Miscellaneous Operations.

Each topic includes a detailed description of the general theory including mathematical formulation application topics, instructions on how to create a CSA file using the EDM software, and detailed instructions on how to setup the CoCo hardware and make a measurement.

This document references the CoCo Basic Users Manual. This separate document gives details on the basic operation of the CoCo hardware and details on basic frequency spectrum measurements including theory, EDM software setup and CoCo operations. We strongly recommend that you read the CoCo Basic Users Manual first before proceeding to this document.

## 2. SWEEPED SINE MEASUREMENTS

This section describes the swept sine measurement capabilities of the CoCo. It includes both theoretical background and application information. The Swept Sine Testing option of CoCo-80 has several unique advantages over similar products in the market, including:

- The measurement channels with very high dynamic range ensure continuous test over high dynamic range UUT (Unit Under Test). It is common to achieve 130~150dB dynamic range with CoCo-80.
- Special tracking filters realized based on TVDFT (Time Variant Discrete Fourier Transform) provide excellent spectrum estimation.
- Special algorithm enables test in wide frequency range. The result of both low and high frequency testing is excellent.
- Time domain signals are always available for viewing and recording.
- Log, Linear sweep modes are available.
- Auto-gain adjustment with closed-loop control capability to prevent input range overloading.

### Sine Signal Used for Testing

Broadband random, sine, step or transient signals are widely used as excitation signals in test and measurement applications. Figure 1 illustrates that an excitation signal  $x$ , can be applied to a UUT (Unit Under Test) and generate one or multiple responses denoted by  $y$ . The relationship between the input and output is known as the transfer function or frequency response function and represented by  $H(y,x)$ . In general a transfer function is a complex function that modifies the input signal magnitude and phase as the excitation frequency changes.



■ Figure 1 Left: a UUT with one response; Right: a UUT with two responses.

With swept sine excitation, the characteristics of the UUT system can be measured experimentally. These characteristics include:

- Frequency Response Function (FRF), which is described by:
- Gain as a function of frequency
- Phase as a function of frequency
- Resonant Frequencies
- Damping factors
- Total Harmonic Distortion
- Non-linearity
- Others

Frequency response can be measured using the FFT, cross power spectral method with broadband random excitation. Broadband excitation can be a true random noise signal with Gaussian distribution, or a pseudo-random signal of which the amplitude distribution can be defined by the user. The term “**broadband**” may be misleading, as a well implemented random excitation signal should be frequency band-limited and controlled by the upper limit of the analysis

frequency range. That is, the excitation need not excite frequencies above that which can be measured by the instrument. The CoCo random generator will only generate random signals up to the analysis frequency range. This will also concentrate the excitation energy on the useful frequency range.

The advantage of using broadband random excitation is that it can excite the whole frequency range in a short period of time so the total testing time is less. The drawback of broadband excitation is that its frequency content is spread over a wide range within a short duration. The energy contribution of the excitation at each frequency point will be much less than the total signal energy (roughly, it is -30~ -50dB less than the total). Even with a large number of averaging in the FRF estimation, the broadband signal will not effectively measure the extreme dynamic characteristics of the UUT.

Swept sine measurements, on the other hand, can optimize the measurement at *each frequency point*. Since the excitation is a sine wave, all of its energy is concentrated at a single frequency, eliminating the dynamic range penalty in a broadband excitation. In addition, if the frequency response magnitude drops, the tracking filter of the response can help to pick up extremely small sine signals. Simply optimizing the input range at each frequency can extend the dynamic range of the measurement to beyond 150 dB.

## Introducing Sweeping Sine

A sine signal with a fixed frequency  $f_0$  can be expressed as:

$$x(t) = \sin(2\pi f_0 t)$$

where t represents time. A sweeping sine signal has a changing frequency that is usually bound by two limits. The frequency change can be either in the linear scale or logarithmic scale based on different user requirements. The swept sine signal can be defined by the following parameters:

- The low frequency boundary, which is simply called Low Frequency or  $f_{Low}$
- The high frequency boundary, which is simply called High Frequency or  $f_{High}$
- The sweeping mode, either logarithmic or linear
- The sweeping speed, in either octave/min if the sweep mode is logarithmic, or in Hz/Sec if the sweeping mode is linear
- The amplitude of the sine signal,  $A(f, t)$ , which can be a constant or a variable of time and frequency.

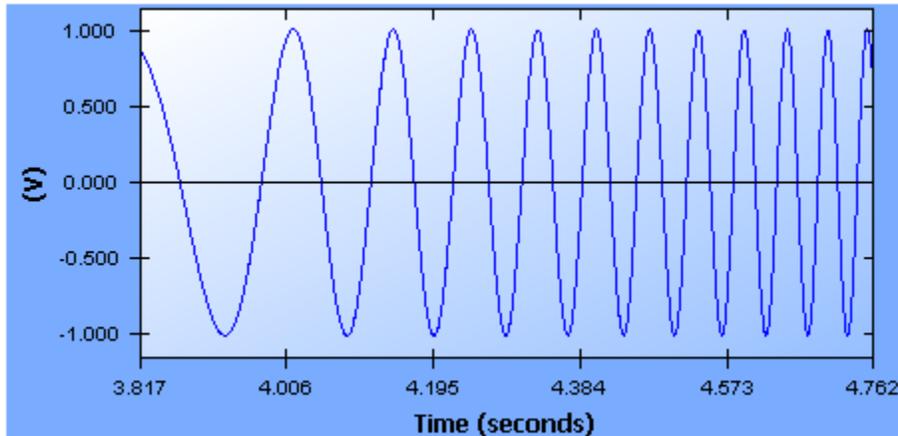
$$x(t) = A(f, t) * \sin(2\pi (f(f_{Low}, f_{High}, Speed)) t)$$

The instantaneous frequency  $f(f_{Low}, f_{High}, Speed)$  represents the current frequency of the sweeping sine. It is a changing variable and usually displayed on the screen as **Sweeping Frequency**.

The sweeping frequency can also be manually controlled during the test with the **Hold, Resume, Jump or Pause controls**.

Unlike some DSA products which use swept sine test with **multiple discrete stepped sine tones in a sequence**, the CI swept sine test uses a true digital synthesizing technique to generate sine sweeps with extreme analog-like smooth transition from one frequency to another. This ensures

that there are no sharp transitions during the test that might “shock” the UUT. The picture below shows a typical swept sine signal with 1.0 Vpk.



■ Figure 2. Typical digitally synthesized swept sine signal.

### **Sweeping Mode: Logarithmic or Linear**

A swept sine can sweep in either linear or logarithmic mode. Linear sweep means the frequency will change at a constant speed, with units of *Hz/sec*. In this case the sweep rate is constant and the same at all frequencies.

Alternatively, the sweeping mode can be set as logarithmic or Log. In Log mode, the sweeping speed is slower at low frequencies and fast at higher frequencies. In Log mode, the sweeping speed units are in *Octave/Min*. *1.0 Oct/min* means that the frequency will take one minute to double from 1kHz to 2kHz, or from 100Hz to 200Hz, or from 0.5Hz to 1.0Hz.

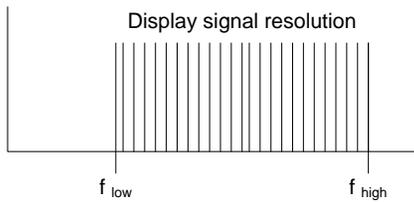
Most testing specifications ask for logarithmic sweeping for two reasons. The first is due to the fact that it takes longer to measure one or multiple sine cycles at low frequency than at high frequency. The second is that most mechanical and electrical systems exhibit characteristics that are better described in logarithmic frequency scale. This is because dynamics such as resonant frequencies occur over large frequencies spans: some at low frequencies and some at high frequencies. If linear sweeping is adopted, you may find that whatever the speed you choose, it is either too slow in the high frequency end or too fast in the low end. With Log sweeping mode, this problem is solved.

On the CoCo, once the sweeping mode is set to either Linear or Log in a test, the frequency distribution of the display signals will be set to linear or logarithmic accordingly. This will be discussed in the following section about the display resolution. The sweeping speed unit will also be set to either Hz/Sec. or Oct/Min automatically.

### **Resolution of Display Signals**

In the CoCo the sweeping sine signal is point-by-point digitally synthesized. It has “infinitely” fine resolution in frequency transition. It does not jump from one frequency to another. The user may wonder how the sweeping signal is displayed. The user first needs to set the size of the displaying signals, say 1024 or 2048. The CoCo will distribute the frequency bins between  $f_{Low}$ ,  $f_{High}$ . In Linear mode, the frequency spacing between two adjacent lines is represented by the frequency

resolution; In Log mode, the frequency spacing between two adjacent lines of the signal will be represented by a ratio.



For example, if a linear sweep is defined with  $f_{Low} = 100\text{Hz}$ ;  $f_{High} = 1000\text{Hz}$ ,  $Signal\ Size = 1024$ , then the first line of the signal will be allocated to 100Hz, the last to 1000Hz. The frequency bins of the signals will be evenly distributed with frequency resolution of

$$(1000-100)/(1024-1) = 0.879765\text{Hz}$$

If a logarithmic sweep is defined with **Log Sweep Mode:**  $f_{Low} = 100\text{Hz}$ ;  $f_{High} = 1000\text{Hz}$ ,  $Signal\ Size = 1024$  Then the first line of the signal will be allocated to 100Hz, the last to 1000Hz. The Frequency resolution will be represented by a ratio as:

$$1000\text{ Hz} = 100\text{ Hz} * \text{ratio}^{(1024-1)}$$

$$\text{ratio} = \left(\frac{1000}{100}\right)^{\frac{1}{1024-1}} = 1.00225335$$

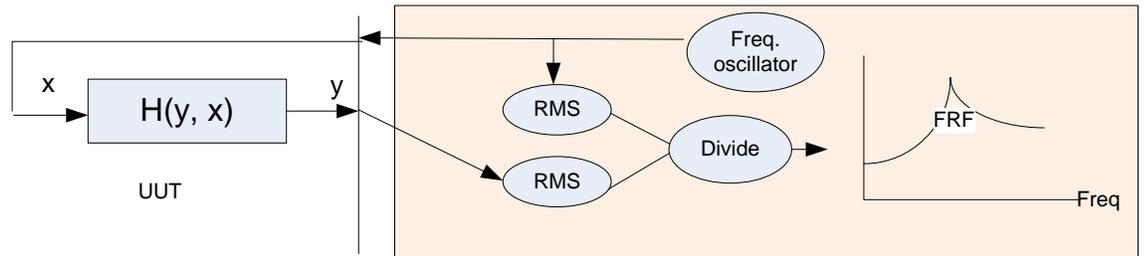
This means that if the first line is at frequency 100Hz, the second line will be at 100.225335Hz, the third at 1.00451178Hz and the 1024th line at 1000Hz.

Once allocated, the display signals will keep the history of each calculated result. The CoCo will update the points that are near the instantaneous frequency of the sweep. This is how the display signals are created. With this design, the user should understand that increasing the resolution of the display signals will not increase or decrease the quality of the swept sine.

## Tracking Filters

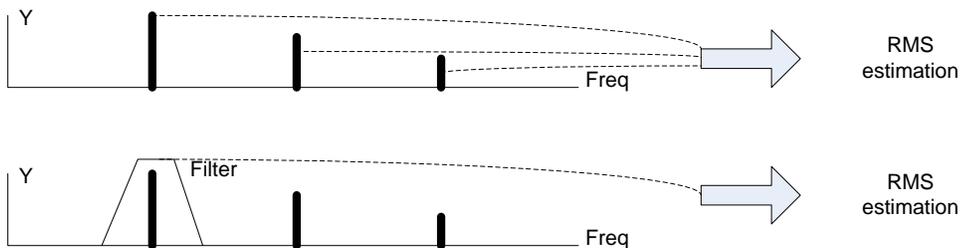
Historically, swept sine tests were originally conducted using analog technology where sine generator and measurement were all implemented in the analog domain. A very simple swept sine testing instrument consists of the following components:

- A sine oscillator of which the frequency can be changed
- An RMS estimator to the output source
- An RMS estimator to the input signal
- A divider that divides the RMS measurements between input and output
- A display or plotter to show the divided results.



■ Figure 3. Analog swept sine implementation.

In many cases the UUT response is not linear. With very pure sine excitation, the response signal may contain strong harmonics. For example with a sine tone excited at 100Hz, the response signal may contain content at 200Hz, 300Hz and so on. A simple RMS estimator will not be able to distinguish the amplitudes at these content therefore the FRF calculation will not be accurate. To overcome this problem, a tracking filter can be applied that centers at the sweeping frequency, and the RMS estimator can be applied to the output of the tracking filter as shown in Figure 4.



■ Figure 4. Tracking filter implementation.

With a filter in place, the RMS estimator will accurately measure the frequency amplitude at the sweeping frequency. The energy at other frequencies will be filtered out.

The challenge of realizing such a filter in the swept sine test is that the filter has to track the center frequency of the sine frequency. Not only does the center frequency of the filter need to change, but also the bandwidth. To give an example, when the sweeping frequency is at 100Hz, it is reasonable to use a filter bandwidth of 50~100Hz. When the sweeping frequency goes down to 10Hz, the next harmonics will be at 20Hz. A filter with bandwidth of 50~100Hz will be too wide to use. To address the problem, a so called the tracking filter is used. Tracking filter changes both its center frequency and bandwidth according to the sweeping frequency. In the analog-made swept sine equipment, this is realized using mixing frequency technology with expensive electronic components. With digital technology, the digitally synthesize tracking filters are implemented in software at no additional hardware cost.

The bandwidth of the tracking filter is a key control parameter. In CoCo, it is defined as a percentage of the sweeping frequency. The user can select a percentage between 100% and 7%. A percentage of 100% means the equivalent bandwidth of the tracking filter is the same as its sweeping frequency. 50% means its bandwidth is  $\frac{1}{2}$  of the sweeping frequency.

CoCo uses a proprietary digital filter that allows very fast response and clean detection of the sine RMS value.

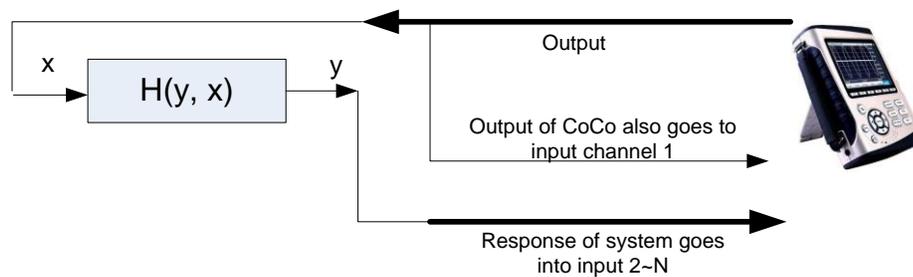
## Measurement Quantities

Measurement quantities that can be monitored during the swept sine test include: time stream of each channel (raw data), spectrum of each channel, frequency responses, coherence, and phase between responses to the reference channel.

**Time streams:** time streams appear the same as any other applications on CoCo. Time streams are always available for viewing and recording. It is a very useful tool to observe whether the input signals are in the valid range. The recorded sine wave can be used for further post-processing. In CoCo, the time streams are often denoted as ch1, ch2 etc.

**Spectra:** The term spectrum is used to refer to the measurement trace in the frequency domain of each channel. It is represented in 0~Peak. The engineering unit of the spectrum is determined by the sensor used by the input channel. The resolution of spectra does not affect the quality of sine wave. In CoCo, the spectra are often denoted as Spec(ch1), Spec(ch2) etc..

**Frequency Response Functions (FRF):** FRF of UUT can be measured using input channel 1 as reference channel and other channels as response channels. The connection should as shown in Figure 5.



■ Figure 5. Frequency response measurement with CoCo.

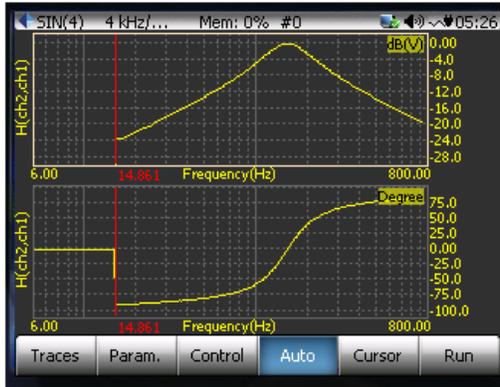
The CoCo will provide the FRF functions of each response channels to the output channel. FRF signals include both phase and magnitude information. In CoCo, the FRF are often denoted as  $H_{yx}$

The number of FRF signals that can be monitored depend on the number of input channels on the CoCo hardware. For example, a CoCo-8, with 4 input channels can monitor 3 FRFs:  $H(\text{ch2}, \text{ch1})$ ,  $H(\text{ch3}, \text{ch1})$  and  $H(\text{ch4}, \text{ch1})$ .

To connect the signal source to the input of UUT and back to input channel 1, you can use a BNC T-connector.

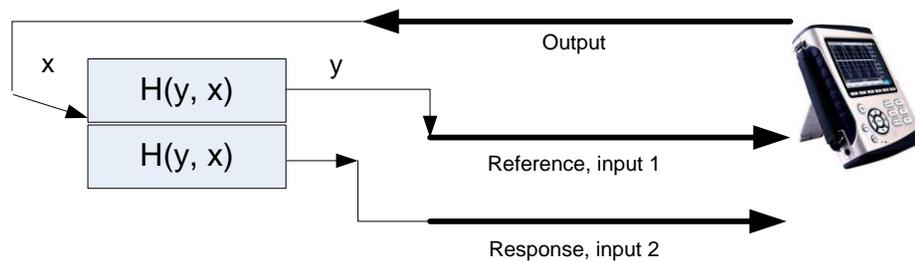


■ Figure 6 BNC T-Connector



■ Figure 7. Frequency response function.

When you measure the ratio between two response channels, it is more accurate to refer to the signals as **Transmissibility functions** instead of FRF because the reference signal is not really the excitation to the UUT. Figure 8 shows how to connect and measure the transmissibility between two response channels.



■ Figure 8. Typical transmissibility measurement.

Transmissibility measurements are used in many applications. For example, it can be used in “back-to-back” transducer calibration where an accurate reference transducer is used to calibrate a less accurate one.

### Output Control Modes

Recall that the sine tone amplitude can be a variable of time and frequency.

$$x(t) = A(f, t) * \sin(2\pi (f(f_{Low}, f_{High}, Speed)) t)$$

We have discussed how the frequency can be changed and controlled by the sweeping mode, sweeping range and sweeping speed. This section discusses how the output amplitude  $A(t)$  is controlled.

There are three Output Modes provided in the CoCo:

- Constant Output Level
- Output Level Profile
- Input Profile with Auto Gain Control

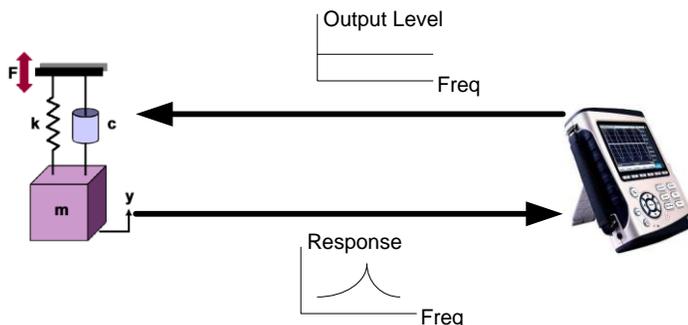
### Constant Output Level

$$A(f, t) = \text{constant}$$

The Constant Output Level is the simplest way to generate the output. It uses a constant output level that is usually defined in the 0-peak volt. For example a 1Vpk means the output swept sine is in 1V in 0-peak.

When the constant output level is used, the response may show peaks or valleys.

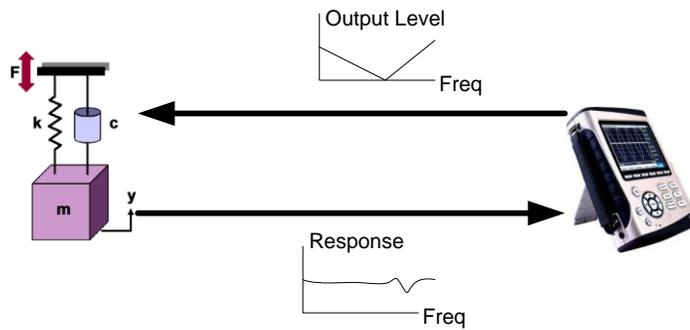
The picture below shows that a constant level sweep is applied to a Single Degree of Freedom (SDOF) device. The voltage output of the CoCo will be converted to force using a mechanical excitation system. We would expect the response measured in either displacement, velocity or acceleration will show a resonant peak.



The drawback of using constant level mode is that sometimes the dynamics of the system vary so extreme that the response signal may exceed the input range. This is very common with systems that have light damping. For example a UUT with 60dB dynamic range, which is quite common, will show the magnitude of the response change 1000 times over the test.

### Output Level Profile

With output level profile control the output level  $A(f, t) = A(f)$  is defined by the user. To overcome the problems with large range of variation of the response, it is possible to attenuate the excitation signal at certain frequency ranges. In the example above, because the resonance frequency is likely known to the operator, we can set the output to a lower level in that specific sweeping frequency range. We call this frequency dependent output level control the Output Level Profile.



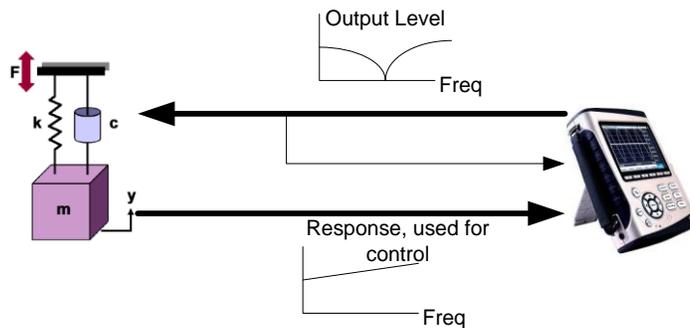
■ Figure 9. Output level profile.

Figure 9 shows that we purposely create a notch in the output level profile so that the response signal is attenuated in the resonance area.

Making the output a frequency dependent signal may help to improve the FRF or transmissibility measurement. It is much better than a constant level output. The drawback of this method is that the UUT dynamics must be known before the test. Another issue is that the output level profile may not be created accurately to match the dynamic characteristics of the UUT. To overcome this difficulty, the CoCo also includes a close-loop control method to allow the auto-gain control.

### Auto Gain Control

With auto gain control,  $A(f, t)$  is calculated in real time based on the target input and close-loop control gain. This advanced method can be explained in Figure 10.



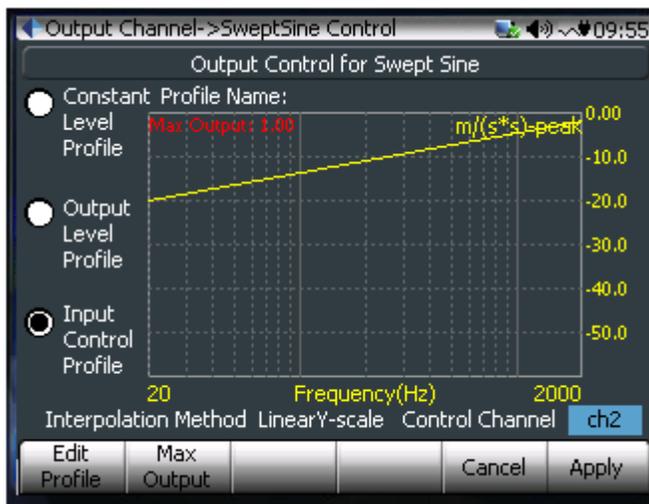
■ Figure 10. Auto gain control mode.

First the user must set up the target profile for one of the response channels (input to the CoCo). The shape of this target profile (Input Profile) does not need to be a straight line. Then during the sweep, the CoCo measures the transfer function between the response and the output. Taking this transfer function into consideration, the CoCo automatically adjusts its output so that the magnitude of the measured input signal matches the input control profile. Because the transfer function changes with frequency, this method requires a close-loop control logarithm.

The input profile with auto-gain control is the most effective way to excite the system. It can maximize the dynamic range of the input channels. However, care must be taken so that the output channel does not get too large and the input channel is saturated, or the output channel gets too small and the input channel reduces to the background noise level.

It must be noticed that the Output and Input Profiles have different engineering units. The Output Profile always has the engineering unit of Vpk for the sine wave. The Input Profile will have the engineering unit of whatever is measured by that channel. For example if the response sensor is a displacement sensor, then the Input Profile will have displacement units, in 0~Peak. If it is an accelerometer, then it will have acceleration units, in 0~Peak.

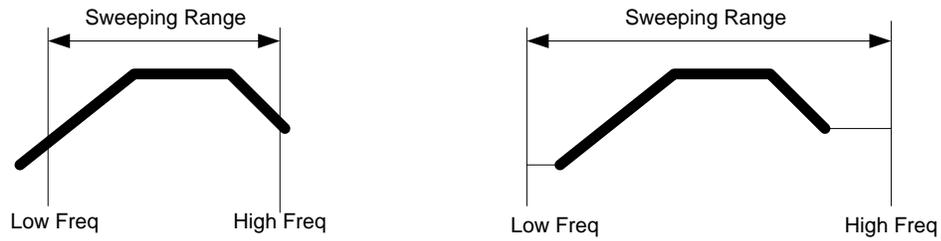
When Input Control Profile is selected, in default we use channel 2 as control channel. You can select any channel other than the reference channel (channel 1) for control.



■ Figure 11. Input auto gain control mode profile.

### **Sweeping Range and Profile**

The sweeping range is controlled by the two boundaries of the frequency range. If the profile setting is not at the same range of the sweeping range then the CoCo will automatically adjust the range.



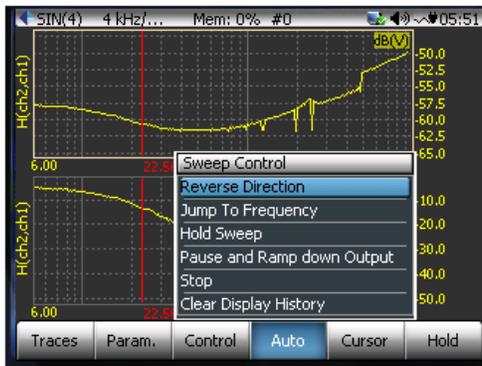
■ Figure 12. Sweeping range and profile.

In Figure 12, the thick line represents the profile. If the Low Freq or High Freq limits do not match the Profile, the software extends the ends to the left and right so there are always valid profile value points when the output sweeps.

## Sweep Control

The swept sine output is controlled by **sweeps**. One sweep indicates that the output will generate the sine frequency from the Low Frequency to the High Frequency, or high to low. In addition the user can control the sweep with manual controls including:

- Start Output
- Stop Output: this action will abort the test
- Reverse Direction
- Jump to Frequency
- Hold Sweep: this action will not ramp down the output voltage amplitude. The frequency will be fixed
- Resume Sweep
- Clear Display History



■ Figure 13. Sweep control options.

To avoid shocking the UUT, a sine output will never start or stop abruptly. Instead, the sine wave amplitude slowly ramps up from zero to the desired level. The ramping rate is defined as dB/sec. a 40dB/sec means the sine wave will ramp up or ramp down for 100 times in magnitude in a second. This is a user-defined advanced value.



## 3. ACOUSTIC DATA ACQUISITION: OCTAVE ANALYSIS

The Acoustics Data Acquisition software option includes Fractional Octave Filter Analysis, Sound Level Meters and Microphone Calibration functions.

The Fractional Octave Filter Analysis applies a bank of real-time octave filters to the input time streams and generates two types of signals at the same time: fractional frequency band signals, i.e., *octave spectra*, and the *RMS time history* of each filter band. The output of each real-time filter bank is in fact a 3D waterfall signal that is arranged in the x-axis as logarithmic frequency and z-axis as time. In the frequency direction, frequency weighting is applied. In the time axis, the time-weighting is applied.

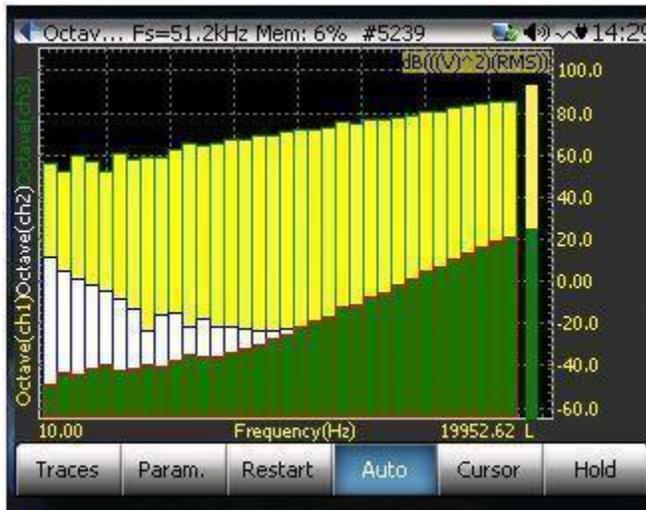
The Sound Level Meter (SLM) is a similar application to octave filters in the acoustic data acquisition. This application is also referred to as an Overall Level Meter. The SLM applies ONE frequency weighting filter to the input signal and time weighting to the output. Various measures are then extracted from both the input and output signals of this frequency weighting filter.

### Fractional Octave Filter Analysis

Acoustics Analysis provides 1/N octave analysis using true real-time digital filters that conform to Conforms to ANSI std. S1.11:2004, Order 3 Type 1-D and IEC 61260-1995 specifications. A, B and C weighting filters can be applied to the input data. Output results are weighted or un-weighted RMS values. The output can be normalized with a calibration value. The results can be plotted on log or linear axes and exact or preferred frequency values are supported.

The Acoustics Analysis provides 1/N octave analysis using true real-time digital filters that conform to ANSI S1.11 and IEC 61260 specifications. Each band filter is designed in accordance to ANSI S1.11 and IEC 61260 specifications by transforming the original analog transfer function to the digital domain by means of the bilinear transform. The filter order can be specified and the frequency ratio can be calculated using the binary or decimal system.

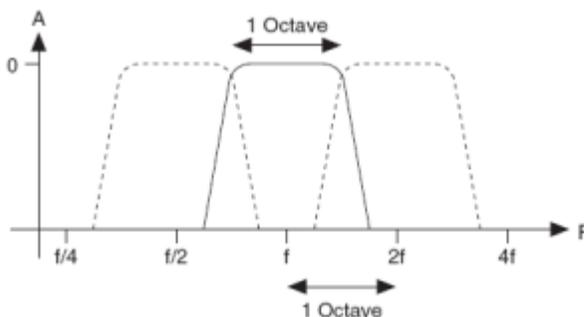
The RMS reading of each octave filter is usually represented by a “bar” in the spectrum plot. Keep in mind that the octave filters have “skirts” on both sides. They are not as straight as the bars depicts. The adjacent filters always overlap. Due to this reason, a sine tone at 1kHz will not only excite the filter with center frequency at 1kHz, but also all other filters. Figure 14 shows how the energy in each band is displayed on the octave spectrum plot using bars.



■ Figure 14. 1/3 octave filter banks.

### Full Octave Filters

An octave is a doubling of frequency. For example, frequencies of 250 Hz and 500 Hz are one octave apart, as are frequencies of 1 kHz and 2 kHz.



■ Figure 15. Full octave filter shape.

Full octave analysis, i.e., 1/1 octave, displays the frequency characteristics of a signal by passing the signal through a bank of band-pass filters where the center frequency of each filter is one octave apart. If the lower and upper cutoff frequencies of a band-pass filter are  $f_L$  and  $f_H$ , then the center frequency,  $f_c$  can be determined with:

$$f_c = \sqrt{f_L * f_H}$$

The nominal frequency ratio G is determined by:

$$G = f_H / f_L$$

Two systems are used in the industry, Base-two or Base-ten systems. For base-two systems,  $G = 2$ . For base-ten systems,  $G = 10^{3/10}$ . Base-ten system is used in CoCo.

The proportional bandwidth property will divide the frequency information uniformly over a log scale. It is very useful in analyzing a variety of natural systems. For example, the human response to noise and vibration is very non-linear and many mechanical systems have a behavior that is best characterized by proportional bandwidth analysis.

### Fractional Octave Filters

To gain finer frequency resolution, the frequency range can be divided into proportional bandwidths that are a fraction of an octave. For example, with 1/3 octave analysis, there are 3 band-pass filters per octave where each center frequency is  $10^{1/10}$  the previous center frequency

In general, for 1/N octave analysis, there are N band pass filters per octave such that:

$$\frac{f_H}{f_L} = (10^{3/10})^{1/N}$$

$$f_{c\ j+1} = f_{c\ j} * (10^{3/10})^{1/N}$$

where 1/N is called the fractional bandwidth resolution.

For CoCo the equation and table below define the center frequency of each fractional filter.

$$f_c = 10^{3X/10N}$$

For example for 1/1 Octave (N=1) the first center frequency (index X = 1) is computed as

$$f_c = 10^{\frac{3 \times (-3)}{10 \times 1}} = 0.125 \text{ Hz}$$

■ Table 1. Octave center frequencies.

	1/1-Octave	1/3-Octave	1/6-Octave	1/12-Octave
Standard	IEC 225-1966 DIN 45651 ANSI S1.11-2004 Order 7 Type 1-D	IEC 225-1966 DIN 45651 ANSI S1.11-2004 Order 3 Type 1-D	N/A	N/A
X (index)	-3 ~ 14	-10 ~ 43	-20 ~ 86	-40 ~ 172
Total number of Filters	18	54	107	213
$f_c$ (Hz)	0.125 – 16k	0.1 – 20k	0.1 – 20k	0.1 – 20k

### Nominal center frequencies (midband frequencies)

Nominal center frequencies are “round” numbers that were inherited from the old analog octave filters. They are rounded midband frequencies for the designation of band pass filters. The nominal midband frequencies for 1/1-octave and 1/3-octave are listed in the ANSI S1.11-2004 Annex A. The standard also describes how to decide the nominal midband frequencies for other fractional octave bands.

The exact center frequency of the filter band is usually not the same as that of nominal frequency. For example, in a 1/3 octave, the exact center frequencies 794.33Hz, 1000Hz and 1258.9Hz are used to correspond to the filters with nominal frequencies 800Hz, 1000Hz and 1250Hz.

### Band Edge Frequencies of Fractional Filters

The low and high edge frequencies of a filter can be calculated based on the frequency ratio, G and the fractional octave resolution N (=1, 3, 6, 12...)

$$\begin{aligned} \text{Lower Edge Frequency } f_L &= f_c * (10^{3/10})^{-1/2N} \\ \text{Upper Edge Frequency } f_H &= f_c * (10^{3/10})^{1/2N} \end{aligned}$$

The bandwidth of the filter is:  $BW = f_H - f_L$

When starting or resetting the filtering operation of the fractional-octave filters, a certain time is required before the measurements are valid. This time is called the *settling time* and is related to the bandwidth of any particular filter. The lowest frequency band has the smallest bandwidth and defines the settling time required before you can consider the complete fractional-octave measurement valid. A good rule of thumb is that the settling time is approximately five divided by the bandwidth.

$$\text{Settling time} = \frac{5}{BW} = \frac{5}{f_H - f_L}$$

Note the settling time depends on the bandwidth which changes with center frequency. A narrower filter and a lower frequency band requires a longer settling time.

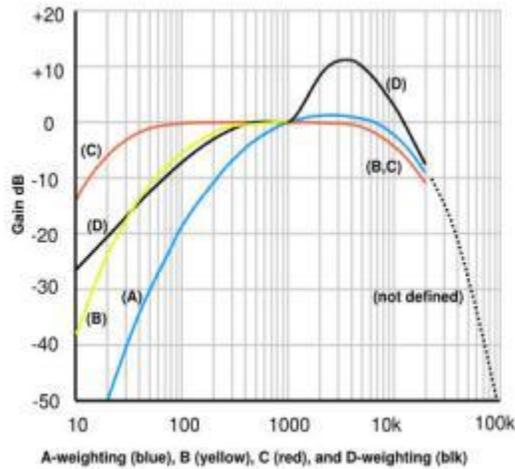
## Analysis Frequency Range

In CoCo, the user can decide the analysis range by changing the lowest and highest  $f_c$  as the Analysis Parameters:

Analysis Range	1/1 Octave	1/3 Octave	1/6 Octave	1/12 Octave
Lowest $f_c$ (Hz)	0.125 1 8	0.1 1 10 100	0.1 1 10 100	0.1 1 10 100
Highest $f_c$ (Hz)	1000 4000 16000	1000 2000 5000 10000 20000	1000 2000 5000 10000 20000	1000 2000 5000 10000 20000

## Frequency Weighting

The human hearing system is more sensitive to some frequencies than others, and its frequency response varies with level. In general, low frequency and high frequency sounds appear to be less loud than mid-frequency sounds, and the effect is more pronounced at low pressure levels, with a flattening of response at high levels. Octave analysis and sound level meters therefore incorporate weighting filters, which reduce the contribution of low and high frequencies to produce a measurement that corresponds approximately to what we hear.



■ Figure 16. Frequency weighting filter shapes.

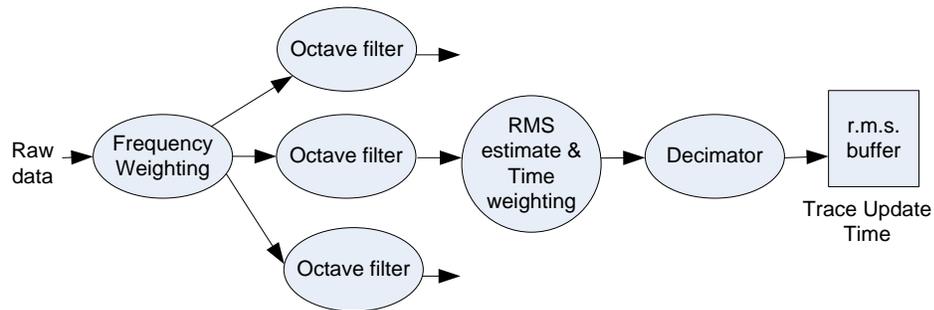
CoCo provides A, C, Z weightings conforming to IEC 61672-1 2002 and B weighting conforming to IEC 60651 in both of Octave analysis and Sound Level Meter. The Frequency weighting in the octave filters will affect the results of all filter bands.

### Time or RPM based RMS Trace of the Octave Filters

The ANSI and IEC standard do not require storing the time history of the band pass filter output. However the user may be interested in viewing this information. On the CoCo the RMS history of all the band pass filters are stored, in the RMS quantity. Below is the description about how the RMSs. history is calculated.

The RMS history can be stored against one of two variables: Time or RPM.

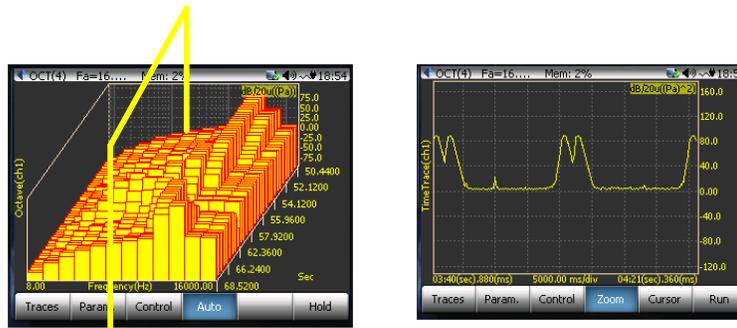
Both the input and output of a digital filter are a series of data points. While it requires excessive memory to keep all the time data of all the filters, it is useful to keep the so called RMS history of each filter output. The RMS time history is computed after the time weighting averaging operation as shown in Figure 17.



■ Figure 17. RMS time history calculation.

The *Decimator* is used so the user can choose the length of time to save the RMS data. For example, given a buffer length of 1024, a *Trace Update Time* of 5 ms will keep about 5 seconds of RMS history; if this update time is set to 5 seconds, it will record 1.4 hours of RMS history.

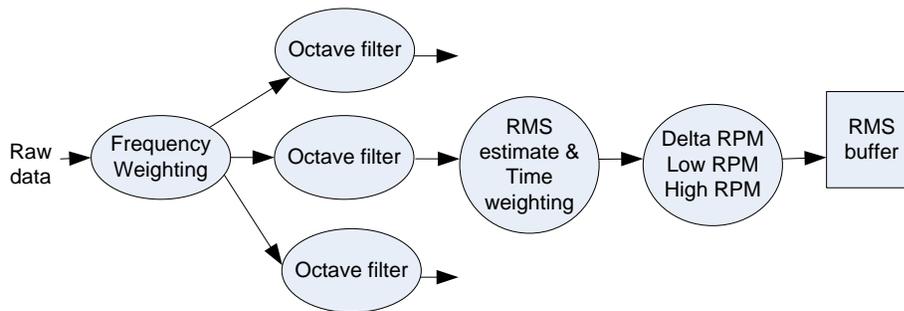
Figure 18 shows the 3D waterfall display of a 1/1 octave filter output. If a *cut* is made in the Z axis direction, the result will be an octave spectrum. If a *cut* is made in the X-axis, the result will be called a *Time Trace*.



■ Figure 18. A cut of 3D Waterfall of octave filter output (left) maps to an RMS time trace (right)

The *Time Trace* stores the history of the RMS of each filter output. The spacing between two points of the *Time Trace* is called *Trace Update Time, in seconds*. On CoCo, one *Time Trace* is allocated for each channel for display. Keep in mind that this buffer of *Time Trace* is the output of a specific filter, the user can change the center frequency of the filter for the *Time Trace* during run time. In the other words, this time trace display buffer will change its content completely when the user switches the *Time Trace Frequency* from one to another.

Alternatively the RMS trace can be stored using RPM as a variable. This method is particularly useful in the automotive NVH applications. The picture below shows how one of the outputs of filters can be stored in RPM trace.



■ Figure 19. Store RPM based RMS traces.

### Exponential and Linear Averaging

**Linear average:** The Linear average method uses a fixed time period to sum up the historical power value of each filter and then takes the square-root to calculate the averaged RMS value. In Linear average, the RMS trace update time is governed by the time period of the averaging. For each time period of averaging, one RMS value per frequency bin is produced.

**Exponential average:** Exponential average applies an exponential time constant to the historical power values of each filter and takes the square-root of the averaged power value. A time constant of 0.125 seconds is equivalent to “Fast” averaging and 1.0 second is equivalent to “Slow”

averaging in the acoustics. In exponential average, the RMS trace update time is independent of the time constant.

Peak Hold: Peak Hold retains the maximum value in each frequency bin over the period of time since last “start” or “restart”.

As we discussed previously, each filter may have different settling time.

### **Measurements available to CoCo in Octave Analysis mode**

The following measurement quantities are available on the CoCo in the octave measurement mode.

#### **Time streams of input channels**

In CoCo, time domain data is always available in the form of long time history. The user can view and record the time signals. The limitation is that the sampling rate of the time signals cannot be arbitrarily changed. It is always set internally by the system based on analysis frequency range, i.e., the highest center frequency of the filter bands.

#### **Octave Spectra**

Each input channel will have an octave spectrum.

#### **RMS Trace**

Each input channel will have one RMS trace to display the RMS history. This RMS is the output of the filter for a specific band. The RMS trace is defined as the Time-RMS trace or RPM-RMS trace at the CSA Editor level. You cannot change between Time and RPM based for a specific CSA.

## 4. ACOUSTIC DATA ACQUISITION: SOUND LEVEL METER

An analog sound level meter measures the sound pressure level. The standard sound level meter is more correctly called an *exponentially averaging sound level meter* as the AC signal from the microphone is converted to DC by a RMS circuit and thus it must have a time-constant of integration. This is referred to as time-weighting. Three of these time-weightings have been standardized, 'S' (1s) originally called Slow, 'F' (125 ms) originally called Fast and 'I' (35 ms) originally called Impulse. The output of the RMS circuit is linear in voltage and is passed through a logarithmic circuit to give a readout in decibels (dB). This is 20 times the base 10 logarithm of the ratio of a given root-mean-square sound pressure to the reference sound pressure. Root-mean-square (RMS) sound pressure is obtained with a standard frequency weighting and standard time weighting.

With the advent of digital technology and increasing accuracy of the electronic circuits, the sound level meter functions are more recently calculated in the digital domain. One of the important factors of such implementation is that the instrument must provide very high dynamic range so that both weak and strong signals can be calculated and observed. CoCo provides 130dB dynamic range. High dynamic range is one of the most important measures of the quality of an acoustic analyzer.

A traditional sound level meter only includes the 1/1 and 1/3 octave filters. In the CoCo system octave analysis is provided in addition to the other analysis functions providing more flexibility and computation power than a traditional sound level meter.

You should use Octave Analysis as the template to create a CSA projects when fractional octave analysis is needed. In both the Octave Analysis and Sound Level Meter templates the user can see the frequency weighted readings (such as dBA). But the reading results may be slightly different when comparing Octave Analysis and Sound Level Meter results. This is because the data processing flow in octave filter analysis and sound level meter is computed differently. In the octave analysis, the dBA, i.e., the A-weighted sound level is computed by applying the frequency weighting function to the output of each individual filter bank; while in SLM, the A-weighted sound level is created by applying the A-weight filter in the entire time domain. The SLM template should be used to obtain the dBA or similar overall readings for most sound studies that might be compared to results taken with a traditional sound level meter because the computation is more similar to that of a traditional sound level meter.

### Terms and Definitions

In this section we will define the terminology used in the SLM software options.

**Reference sound pressure** It is conventionally chosen as 20  $\mu\text{Pa}$ . This is the threshold of hearing of the average person and is used to compute the sound pressure level in the dB scale.

**Sound pressure level (in dB)**

Sound pressure level (dB) is defined as twenty times the logarithm to the base ten of the ratio of the RMS of a given sound pressure to the reference sound pressure. Sound pressure level is expressed in decibels (dB); symbol  $L_p$ .

### Peak sound pressure

The peak sound pressure is the greatest absolute instantaneous sound pressure during a stated time interval.

### Peak sound level (in dB)

The peak sound level (dB) is defined as twenty times the logarithm to the base ten of the ratio of a peak sound pressure to the reference sound pressure, peak sound pressure being obtained with a standard frequency weighting. (example letter symbols are  $L_{peak}$ ,  $L_{Cpeak}$ )

### Frequency weighting

Frequency weighting is the difference between the level (dB) of the signal indicated on the display device and the corresponding level of a constant-amplitude steady-state sinusoidal input signal, specified in the IEC or ISO standards as a function of frequency. It accounts for A, B and Z frequency weighting discussed in the previous section.

### Time weighting

Time weighting is an exponential function of time, of a specified time constant, that weights the square of the instantaneous sound pressure. This is the same as exponential averaging in the time domain to the instantaneous sound pressure.

It is a continuous averaging process that applies to the output of a frequency weighting filter or one of the fractional filters. The amount of weight given to past data as compared to current data depends on the exponential time constant. In exponential averaging, the averaging process continues indefinitely.

In a sound level meter the time weighting exponential averaging mode supports the following time constants:

**Slow** uses a time constant of 1,000 ms. Slow averaging is useful for tracking the sound pressure levels of signals with sound pressure levels that vary slowly.

**Fast** uses a time constant of 125 ms. Fast averaging is useful for tracking the sound pressure of signals with sound pressure levels that vary quickly.

**Impulse** uses a time constant of 35 ms if the signal is rising and 1,500 ms if the signal is falling. Impulse averaging is useful for tracking sudden increases in the sound pressure level and recording the increases so that you have a record of the changes.

**User Defined** allows you to specify a time constant suitable for your particular application.

### Time-weighted sound level (in dB)

This is twenty times the logarithm to the base ten of the ratio of a given RMS sound pressure to the reference sound pressure, RMS sound pressure being obtained with a standard frequency weighting and standard time weighting. (example letter symbols are  $L_{AF}$ ,  $L_{AS}$ ,  $L_{CF}$ ,  $L_{CS}$ )

### Maximum and minimum time-weighted sound level (in dB)

This is the greatest and lowest time-weighted sound level within a stated time interval. (example letter symbols are  $L_{AFmax}$ ,  $L_{ASmax}$ ,  $L_{CFmax}$ ,  $L_{CSmax}$ ,  $L_{AFmin}$ ,  $L_{ASmin}$ ,  $L_{CFmin}$ ,  $L_{CSmin}$ )

### Time-average sound level (equivalent continuous sound level) (in dB)

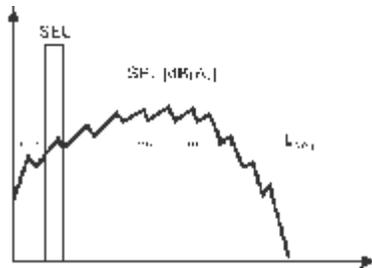
This is twenty times the logarithm to the base ten of the ratio of a RMS sound pressure during a stated time interval to the reference sound pressure, sound pressure being obtained with a standard frequency weighting. (example letter symbols are  $L_{Aeq}$ ,  $L_{Ceq}$ )

**Sound exposure**

This is the time integral of the square of sound pressure over a stated time interval or event. Sound exposure is used to measure high-level, short duration noises and to study their effects on humans.

**Sound exposure level (in dB)**

Sound exposure level is the total sound energy of a single sound event and takes into account both its intensity and duration. Sound exposure level is the sound level you would experience if all of the sound energy of a sound event occurred in one second. This normalization to duration of one second allows the direct comparison of sounds of different durations.



■ Figure 20. Sound exposure level illustration.

Figure 20 shows the relationship between the Sound Exposure Level (SEL), the Sound Pressure Level (SPL), and the Leq. The Leq is the constant level needed to produce the same amount of energy as the actual varying sound (the SPL).

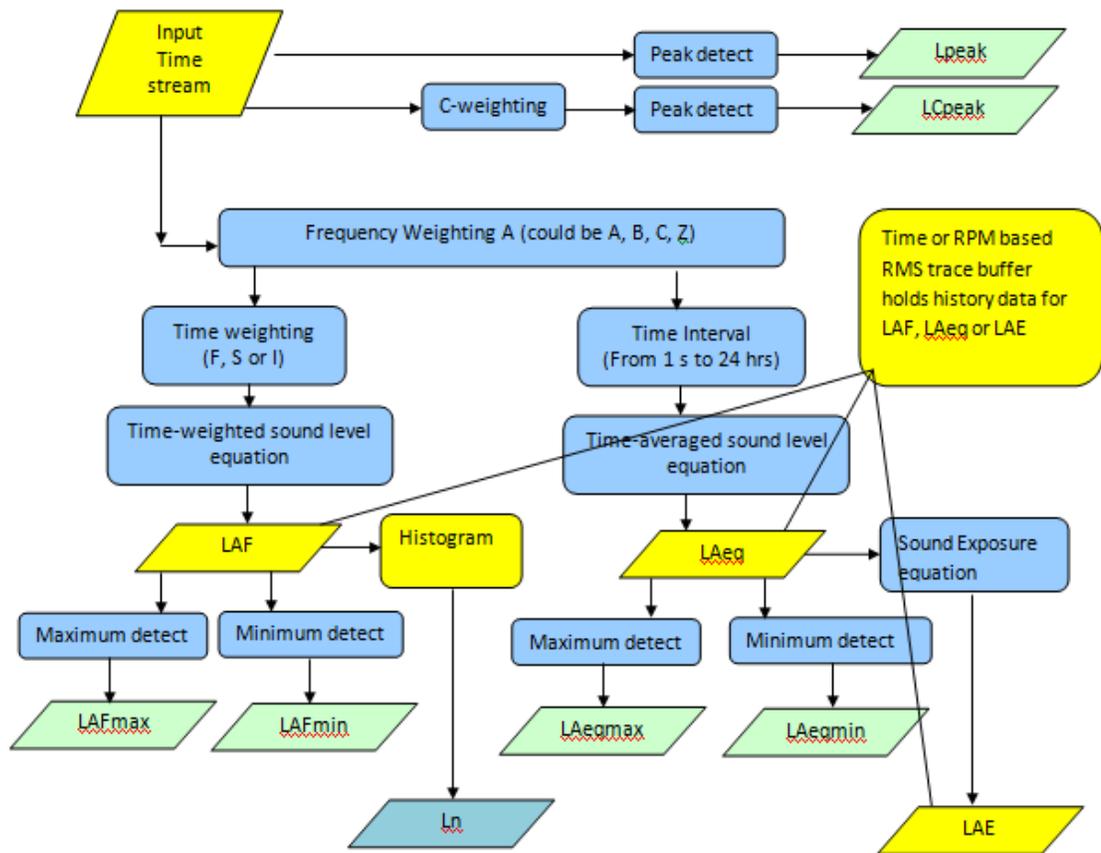
The SEL is the Leq normalized to 1 second. It is what the Leq would be if the event occurred over a one second duration.

**Statistical Level (LN)**

LN is defined as the sound pressure level which is exceeded N% of the time over the duration of a measuring time interval. L0 is the maximum level over the duration of the measurement. L100 is the minimum.

**Data Processing Diagram**

Figure 21 shows the data processing diagram for ONE input channel for all the SLM measurements when A-weight is applied.



■ Figure 21. Sound level meter computation diagram.

In the SLM measurement, after the digitized data comes in, it is split into three paths: one goes to frequency weighting A, B, C or Z and one goes to C weighting or no weighting. The peak detection is computed from the output of C weighting or no weighting. The output of frequency weighting (A, B, C or Z) is further split into two paths. The first will go to a *time weighting* function which is more or less equivalent to an exponential averaging mode to calculate LAF; the second path goes to a time averaging function, which is equivalent to a linear averaging mode to calculate Leq.

With A-weighted applied as shown in the example, the list of symbol to be used by this instrument is:

Symbol of Measured Values	Description
LAF	A-weighted, F time-weighted sound level
LAFmax	Maximum A-weighted, F time-weighted sound level
LAFmin	Minimum A-weighted, F time-weighted sound level
LCpeak	Peak C sound level, greatest absolute instantaneous C-weighted sound pressure level
Lpeak	Peak sound level, greatest absolute instantaneous sound pressure level
LAeq	A-weighted, time-average sound level (equivalent continuous sound level)
LAeqmax	Maximum A-weighted, time-average sound level (equivalent continuous sound level)
LAeqmin	Minimum A-weighted, time-average sound level (equivalent continuous sound level)
LAE	A-weighted sound exposure level
$L_N$ (N = any integer between 0~100)	Statistical Level general term
L1, L5, L50, L95....	Statistical Levels with specific N values. The sound level exceeds this level 1, 5, 50 or 95 percent of the time for the duration of the measurement.

### Measurements available to CoCo in SLM mode

There are two ways to view sound level measurements: instantaneous SLM measures and RMS history. Instantaneous SLM measures represent the most current value of the measurement. RMS history not only shows the most current value, but also a record of historical values against time or RPM. Some of the measures allow only instantaneous values others allow both.

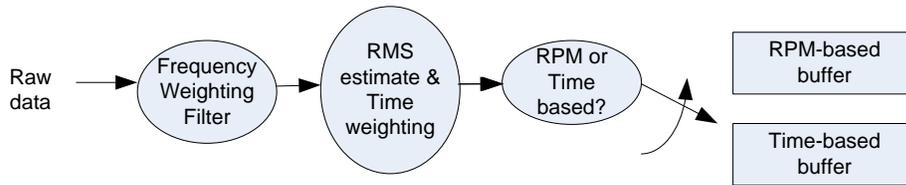
The following measurement quantities are available to CoCo in the measurement.

#### SLM Measures

The following SLM measures are available for real-time reading and can be saved as a data structure for future review.

#### Time Weighted Sound Levels

In CoCo, time weighted sound level is the output of frequency-weighting and then time weighting filters. Time weighting serves an exponential averaging operator. The computation is illustrated in Figure 22.



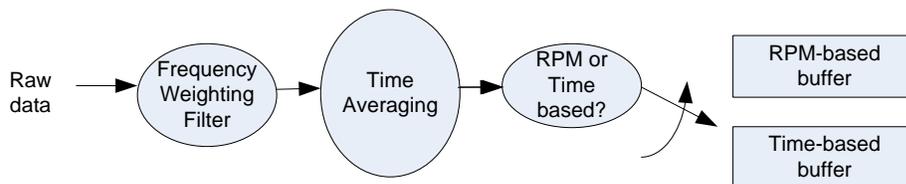
■ Figure 22. Time weighting sound level computation and storage against RPM or Time

The table below shows the symbols for the time-weighted sound level.

Symbol used for time weighted value		Frequency Weighting			
		Z	A	B	C
Time Weighting	F(Fast)	$L_{ZF}$	$L_{AF}$	$L_{BF}$	$L_{CF}$
	S(Slow)	$L_{ZS}$	$L_{AS}$	$L_{BS}$	$L_{CS}$
	I(Impulse)	$L_{ZI}$	$L_{AI}$	$L_{BI}$	$L_{CI}$
	Custom	$L_{ZC}$	$L_{AC}$	$L_{BC}$	$L_{CC}$

### Time Averaged Sound Levels

In CoCo, time averaged sound level is the output of frequency-weighting and then time average operation. Time average serves a linear averaging operator. Figure 23 illustrates the computation.



■ Figure 23. Time averaged sound level computation.

The table below shows the symbols for time-average sound level. In the time averaging sound level measurement, Frequency weighting can be selected as A, B, C or Z. The time interval for time averaging can be set to any value between 1 second and 24 hours.

Frequency Weighting	Z	A	B	C
Symbol	$L_{eq}$	$L_{Aeq}$	$L_{Beq}$	$L_{Ceq}$

### Peak sound level

Only C-weighted and un-weighted are available for peak sound level. This is required by the standards.

Symbol	$L_{peak}$	$L_{Cpeak}$
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### Sound exposure level

Sound exposure level and time-average sound level have the same frequency weighting and same time interval.

Frequency Weighting	Z	A	B	C
Symbol	$L_E$	$L_{AE}$	$L_{BE}$	$L_{CE}$

### Statistical level: value reading

Any statistical level  $L_N$  is the sound level which is exceeded for N% of the defined measurement duration.

Symbols for $L_N$ , N = 1, 5, 50, 95	L1	L5	L50	L95
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### Input Channel Time Streams

In CoCo, time domain data is always available in the form of long time history. The user can view and record the time signals. The limitation is that the sampling rate of the time signals cannot be arbitrarily changed. It is always set internally by the system based on analysis frequency range.

### RMS trace of weighted level, time averaged level or sound exposure

CoCo records an RMS trace of the sound level. The user must choose between the time weighted level  $L_{AF}$ , the equivalent time averaged level  $L_{AEQ}$  or sound exposure level  $L_{AE}$ . Only one can be recorded at a time.

The RMS trace must be selected using one of Time or RMS as variable at the CSA Editor stage.

### Histogram of Time Weighting

CoCo also records a signal containing a histogram of the dB values of the time weighted signal. This signal is used to compute the  $L_n$  data.

## 5. ORDER TRACKING

Order Tracking is a general term describing a collection of software functions used for analyzing the mechanical dynamic behavior of rotating or reciprocating machinery for which the rotational speed can change over time. Unlike the power spectrum and other frequency-domain analysis standards where the changing variable is the frequency, Order Tracking functions present the data related to the variable rotating speed, i.e., RPM (revolution per minute).

The most useful measurements are order spectra and order tracks. An order spectrum gives the amplitude of the signal as a function of harmonic order of the rotation frequency. This means that a harmonic or sub-harmonic order component remains in the same analysis line independent of the speed of the machine.

The technique that observes the changes of any quantity vs. RPM is called **tracking**, as the rotation frequency is being tracked and used for analysis. Most of the dynamic forces exciting a machine are related to the rotation frequency so interpretation and diagnosis can thus be greatly simplified by use of order analysis.

Order tracks are simply the observations to the amplitude of the components with fundamental frequency or harmonics. It is one typical type of *tracking*. There are other types of tracking. For example, the user can track the FFT-based PSD spectra, a fixed band or an octave band etc.; all these can be called tracking.

With the CI Order Tracking package, the instrument can:

- Process a tachometer signal and give a high fidelity RPM measurement
- Measure the order spectra
- Measure the order tracks
- Measure the RPM FFT spectrum
- Measure the energy in fixed bands vs. RPM
- Measure the amplitude and phase of an order relative to the tachometer.

There are several different applications for order tracking. A discussion of some is given below.

The first application, often referred to Run Up/Run Down, is used to evaluate the noise or vibration dynamic response when RPM is used as a changing variable. In this case, the RPM range can be very large, from a few RPM to 10,000 RPM. Typical application tests are used in the automotive or aircraft engines. The measurements can be any physical quantities such as sound, displacement, velocity, acceleration, torque, etc. The analysis measure can be the amplitude or the power of an order, the energy over a fixed frequency band, a bin of octave filter, etc. The phase information of the responses to tachometer is less important in this type of application. In fact the rotating element might be hidden inside of mechanical system. The primary result for this type of measurement is the magnitude of the responses vs. RPM.

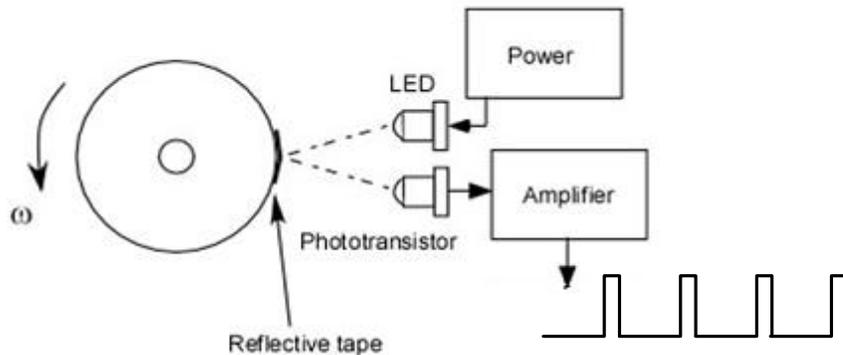
The second application is rotating machine analysis that focuses on the measurement of displacement or velocity of the rotors while it is rotating. The instrument measures the amplitudes of specific orders and their relative phase to a reference signal. The phase is calculated relative to the tachometer input or a separate reference input. This application is common for machine diagnosis and balancing. In this case, the RPM is stable or quasi-stable. Order tracking technology is useful to increase the accuracy of the estimation of orders.

Order signals with phase are useful in the test of rotating machine in the Run Up/Run Down process. This is often presented as a “Bode Plot”. The Bode Plot is a borrowed concept from control theory; it is a collection of Amplitude and Phase data over a changing speed range (i.e. Run Up or Coast Down). Some of the setup information depends on the rate of change of the RPM. The Run Up or Coast Down could take anywhere from a few minutes to a few hours (such as for a cold startup on a turbine). Other displays such as the orbit plot is useful as well.

The CoCo includes the ability to measure RPM based octave analysis and sound levels. This feature is similar to order tracks except that spectra are recorded in octave bands with A, B, C or Z frequency weighting. This feature is included in the Acoustic Analysis and Sound Level Meter CSA Templates instead of the Order Tracking Template. Refer to these sections for more details.

## Tachometer Signal Processing and RPM Measurement

A **tachometer (tacho)** converts the angular velocity of a rotating shaft into an electrical signal, typically a voltage. It is common for calibrated instruments to provide a measurement of the shaft in units of revolutions per minute (RPM) or revolutions per second (RPS). Many modern rotating machines (electric motors, generators, pumps, turbines, IC engines, etc.) have integrated tachometers that can measure shaft angular velocity. An example of an optical tachometer is shown in the figure below.



■ Figure 24. Optical tachometer setup.

The goal of tacho signal processing is to get a clean and stable RPM reading. The tacho signal must be carefully processed to provide a base of tracking. Any order tracking results can only be thought of as being as accurate as the tachometer signals that were used to estimate the instantaneous frequency of the order in the analysis process. If the quality of the tachometer channel is poor, the results from all other channels will be poor or even unreliable.

In old analog methods, tacho channels were conditioned with a tracking ratio tuner with a phase lock loop. The disadvantage of this method is the limited slew rate and the use of complex hardware. To overcome these limitations various digital tacho processing methods have been developed.

From hardware design point, there are two ways to implement a tacho input channel: use a dedicated tacho channel with a digital counter, or use an analog input channel.

### Dedicated tacho channel using counter

Using a dedicated tacho channel, usually without A/D converter, has been popular. This hardware approach contains its own tacho clock which runs at a much higher speed; typically in MHz. This tacho hardware also contains special counters which maintain a continuous counter reading to avoid skipping any triggered cycles of the tacho signal. There is also an option to allow these counters to "average" several tacho periods for cases when the input tacho frequency is very high.

### Using Analog Input Channel

Alternatively, some systems use an analog input data channel as a tacho channel. In this case, the tacho clock is actually the sample rate of the data channels. This sample rate usually limits the tacho frequency range since the tacho range is now set by the input data frequency range requirement. In addition, due to the "frame processing" nature of some not-so-well designed input sampling processes, some instruments may be limited to how they acquire the tacho signal. This restriction usually means they get several tacho cycles in every data frame. The result is often an "averaged" value which is okay unless the tacho signal is changing frequency during the data frame event, which is often the case.

With the advances of electronics and lower cost of electronic components cost is less of a concern. The approach of dedicated tacho channel with a digital counter, without A/D, may or may not be the best choice.

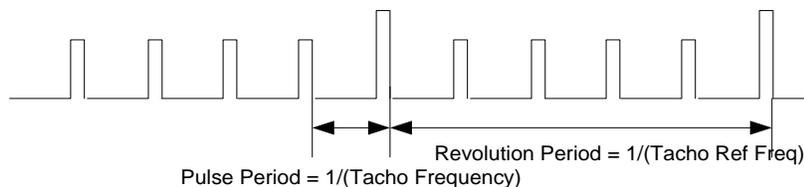
The CoCo-80/90 can use any of the data channel as tacho channel. For simple interface design, usually channel 1 is used for the tacho. While the data input channel is used as a tacho measurement, the special hardware circuitry allows this data channel to sample at the highest possible sampling rate. In the other words, the accuracy of tacho speed measurement is depending on the current range of the analysis frequency. This technique has several obvious advantages:

- The time domain signal of the tacho input is transformed by A/D converter into a digital signal. The user can observe the pulse trains of the tacho signal and set threshold arbitrarily.
- Accurate phase information can be obtained relative to each data channel because the tacho channel, which is fed by high frequency sampling counter, is synchronized with data channels.
- The RPM estimation is not influenced by the current data sampling rate.

### High Pulse per Rev

**Pulse per Rev** is defined as the number of pulses per revolution. Pulse per Rev. must be defined by the user so the instrument can calculate the *reference frequency* of tacho using *tacho frequency*. The relationship is:

$$\text{Tacho Reference freq} = \text{Tacho freq} / \text{Pulses per Rev}$$



In most rotor tests, especially in balancing, the *Pulses per Rev* is simply 1. However, in other cases, such as in flywheel or geared data measurement, the *Pulses per Rev* can be as high as

hundreds. To deal with this situation, a dedicated tacho channel with high speed counter might work better.

In the CoCo-90, in addition to using any data channel as tacho input, a dedicated tacho channel is installed to measure a high speed RPM, or deal with high Pulses/Rev or digital TTL trigger. The counter speed is about 25MHz. This second choice provides a more versatile solution to the user for their applications.

The special tacho hardware design on the CoCo system with the Order Tracking package offers the most accurate possible approach for performing a wide range of Real-Time machinery-related vibration and noise analysis.

### **Pulse Detection**

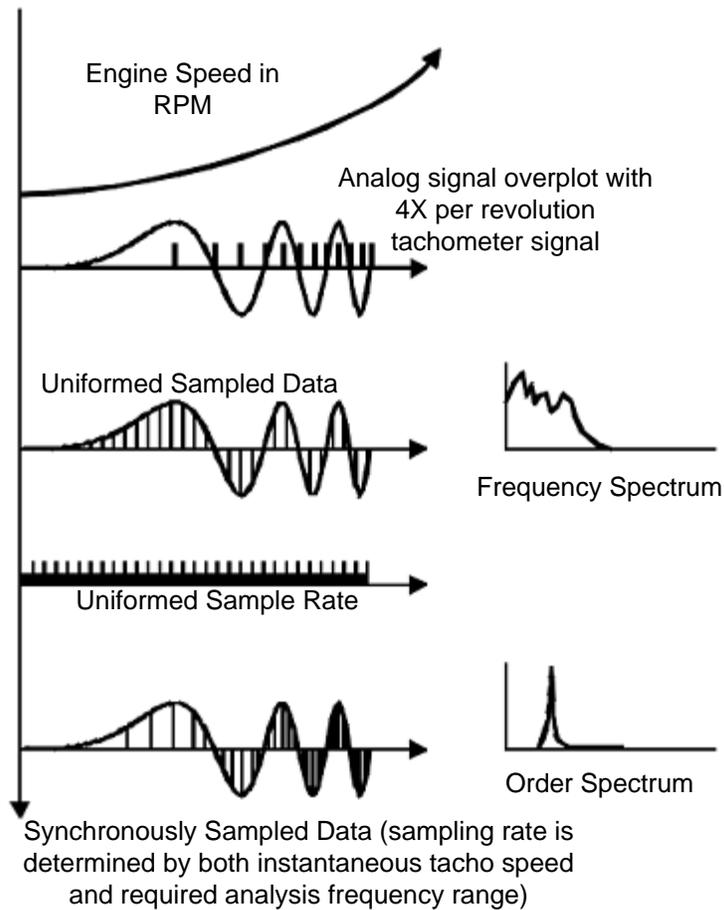
A good tacho processing device should allow the user to see the tacho signal in its original time waveform visually, and set the *Pulse per Rev.*, the threshold of pulse detection. This will help setup the tachometer and diagnose any problems quickly. In the CoCo hardware, a special display window is created so the user can switch between the RPM trace and the tacho original time waveform displays conveniently. The pulse detection threshold can easily be controlled by using Up/Down buttons.

## **Order Tracks and Order Spectrum**

Knowledge of the rotating speed allows presentation of measurement results in the angle and order domains, corresponding to the time and frequency domains. *An order is a frequency normalized with some reference frequency, e.g. the shaft frequency.* This means that the order of a vibration component in the order spectrum indicates the number of vibration cycles per shaft revolution. The magnitude, which can be measured using  $EU_{pk}$ ,  $EU_{rms}$  or  $EU_{rms}^2$ , of an order is the measurement extracted through a tracking filter with the center frequency located at this frequency. Multiple measurements of a range of orders will construct a so called *Order Spectrum*. An order power spectrum measurement gives a quantitative description of the amplitude, or power, of the orders in a signal. It provides a good view of all order components of a signal. This can help you find significant orders and compare the level of different order components

There are two methods to perform rotationally coherent sampling, phase-locked frequency multipliers and digital re-sampling. Phase-locked frequency multipliers were mainly used in early work. They generate sampling pulses based on a rotational reference signal. These sampling pulses control the sampling process. Note that the sampling frequency will depend on the rotational speed, and thus an adjustable anti-aliasing filter is needed. This complicates the method considerably. In the digital re-sampling technique, the time signal is conventionally sampled together with some rotational reference signal. The time signal is then digitally re-sampled to the angle domain by interpolation techniques. The rotational reference signal can be acquired with a tachometer or an incremental pulse encoder.

The following picture shows conceptually how angle data re-sampling can be used to analyze vibrations from an engine during start up. Once the signal has been transformed into its angle domain, the FFT can be applied to analyze the order spectrum of the vibrations.

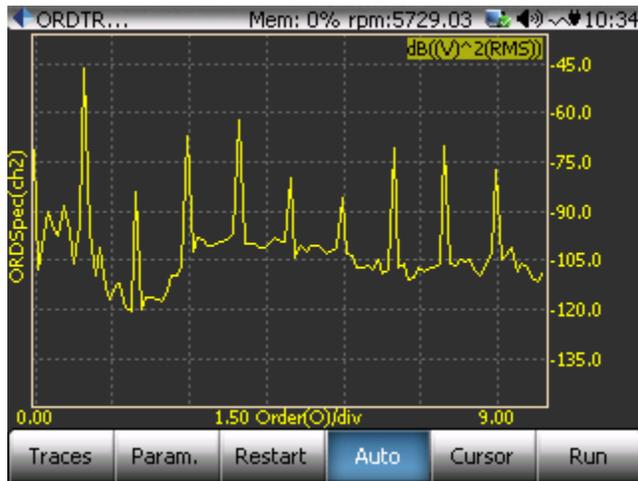


■ Figure 25 Angular Data Re-sampling of a Chip Signal

The last plot in the picture shows that the sampling rate will be determined by both instantaneous tacho speed and required analysis frequency range.

In the CoCo system, the order tracks and order spectrum are computed with a proprietary technology that combines digital re-sampling, data decimation, and interpolation, DFT and FFT calculations.

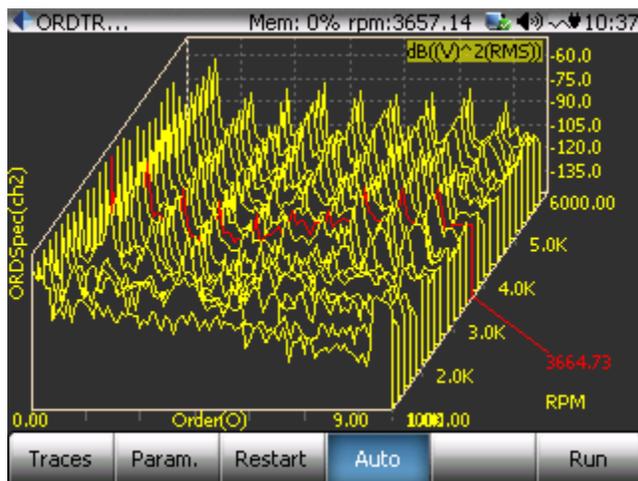
Three measurements can be generated from order tracking computation. The 3D RPM Order Spectrum is simply the a 3-dimensional view of the other two types of measurement.. Another way to visualize these types of plots is that the order spectrum is a cross section of the 3D plot along a fixed RPM value while the order track is a cross section along a fixed order number. The relationship of them is:



■ Figure 26. Typical order spectrum.



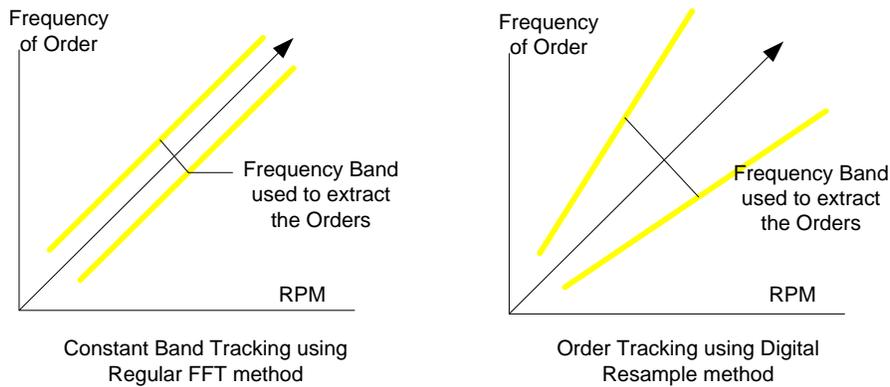
■ Figure 27. Typical order tracks.



■ Figure 28. Typical 3D order track waterfall plot.

An important concept that must be introduced now is called delta order,  $\Delta Order$ . In the FFT based frequency spectrum analysis, the frequency span and frequency resolution are fixed. The capability of discriminating frequency components is equal in both low and high frequency. In rotating machine analysis, we want to have better analysis resolution in the low frequency than that in high frequency. For example, if the rotating speed is at 60 RPM, we definitely care if the instrument can tell the difference between 1Hz (order 1) and 2Hz (order 2); on the contrary, if the rotating speed is at 6000 RPM, the user probably won't care if the instrument can discriminate the measurement between 100Hz (order 1) and 101Hz.

With the digital re-sampling technique, the order tracks and order spectrum are extracted based on a filter with equal  $\Delta Order$  instead of equal  $\Delta Frequency$ . The concept is illustrated in the following figure:

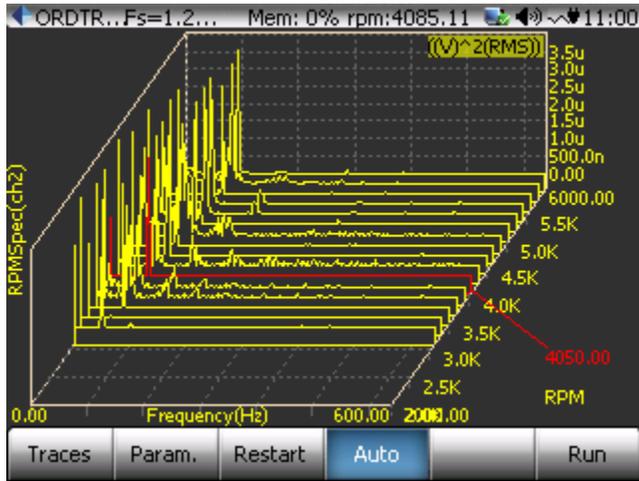


■ Figure 29. Comparison of constant band tracking and digital re-sampling method.

In this figure, the left side shows when the order tracks are extracted using conventional FFT method with fixed resolution, the  $\Delta Frequency$  of tracking filter will be fixed; the right side shows that if the order tracks are extracted using digital re-sampling, the  $\Delta Frequency$  tracking filter will be increased proportionally with the RPM. Obviously, the method of digital re-sampling is more desirable in extracting the measurement of orders.

## RPM Frequency Spectrum

While the order tracks and order spectra are developed to analyze the characteristics of the system on the order space, the measures of fixed bands are also helpful for analysis. Similar to the RMS time trace for a given frequency band with time as variable, the RMS trace can be extracted for a given frequency band with RPM as the independent variable. This is simply called an RPM Spectrum. An RPM Spectrum can be described as a 3D waterfall as shown below:



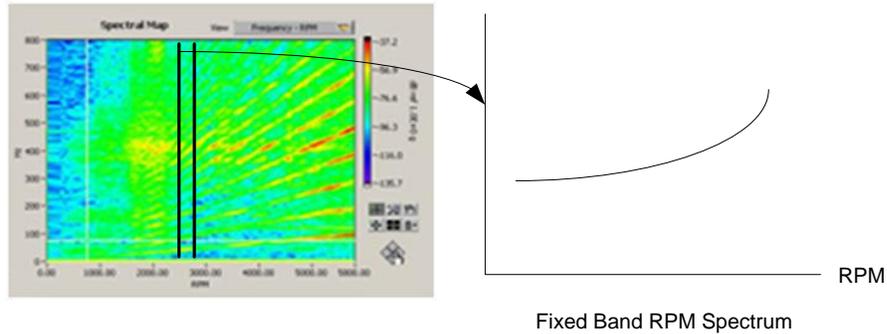
■ Figure 30 RPM spectrum.

The horizontal axis of the 3D RPM Spectrum is frequency. The Z axis is RPM and the measurement unit is usually  $EU_{rms}^2$  or  $EU_{rms}$ . A color map can also be used to describe the magnitude of the whole range as shown below.



■ Figure 31 Color map of an RPM spectrum.

With a 3D RPM frequency spectrum, the instrument can extract the total energy over a fixed frequency band, and plot it with RPM as the independent variable. This is called *Fixed Band RPM Spectrum* as shown below.



■ Figure 32 Fixed band RPM spectrum.

The measurement engineering unit of Fixed Band RPM Spectrum is  $EU_{rms}^2$  or  $EU_{rms}$  representing the total power in a fixed band measured versus rotating speed change. This data is particularly useful to watch the total magnitude in a resonance area when the rotating speed of the shaft is changing. You can define the frequency band around the resonant frequency and perform a run up/down test. Both order tracking and order spectrum cannot extract the response magnitude of the resonance as accurately as a fixed band RPM spectrum because the bandwidth of the tracking filter of order tracking is not explicitly controlled.

## Overall Level Measurement

In order tracking, about it is important to monitor the overall RMS level or power level of the measurement versus RPM. The overall level is a good reference for comparing with other signals such as order tracks or fixed band RPM spectrum.

Overall level can be in unit of RMS ( $EU_{rms}$ ) or power ( $EU_{rms}^2$ ). The horizontal axis is RPM. Below is a typical overall level display.



■ Figure 33. Overall RMS level plot.

## Raw Data Time Streams

In many other order tracking software products, the user can either conduct real-time order tracking analysis, or record the data with other tools and then post process the order tracks, but not both at the same time. The CoCo is a high performance data recorder in addition to a real time analyzer and can do both at the same time. Continuous time streams of each input channels are always available even while order track data is computed..

## Order Tracks with Phase

### The Phase in Rotating Machine Analysis

Many mechanical faults are associated with certain orders, analyzing order magnitude and phase can help you detect mechanical faults directly. For example, a strong first order magnitude indicates imbalance in most cases. Analyzing the first order magnitude can help you identify the imbalance. Moreover, the magnitude and phase of the first order can help you correct the imbalance by adding weights on the appropriate rotor positions. However to fix such an imbalance problem requires phase information of order tracks. A list of the sources of vibration in the rotating machine are:

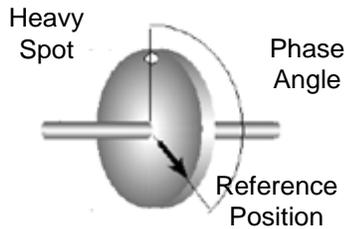
Order	Source of Problem
0.05X~0.35X	Diffuser Stall
0.43X~0.49X	Instability
0.5X	Rubbing
0.65X~0.95X	Impeller Stall
1X	Imbalance
1X+2X	Misalignment
(#Vane)X	Vane/Volute gap
(#Blades)X	Blade/Diffuser Gap

As previously discussed, an order track is the measurement taken for an order, i.e., normalized frequency, versus RPM. In most of the applications of engine related test, the phase information of order tracks are not important. In rotating machine analysis, the phase of the signal is vitally important.

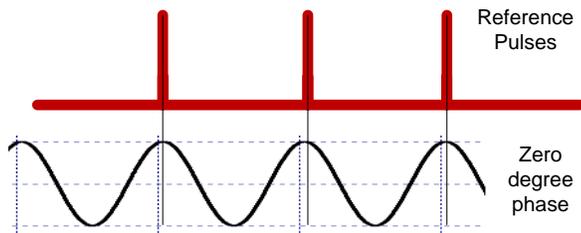
Phase is a relative measurement quantity and can only be measured with a pair of signals. It indicates the time delay at certain frequency between two signals. The phase value can be translated into the difference of relative angle, relative position or propagation time if additional information is given. When we refer to the phase information of one signal, we imply its phase is relative to a reference signal that was mentioned in context.

In rotating machine analysis, the phase of the first order of the rotor can be directly mapped to an angular difference between a signal and the reference signal. The reference signal can be another channel of measurement, or the tachometer signal. The phase difference between two waveforms is often called a phase shift or phase delay. A phase shift of 360 degrees is a time delay of one cycle, or one period of the wave, which actually amounts to no phase shift at all. A phase shift of 90 degrees is a shift of 1/4 of the period of the wave, etc. Phase shift may be considered positive or negative, i.e., one waveform may be delayed relative to another one, or one waveform may be advanced relative to another one. These conditions are called phase lag and phase lead respectively.

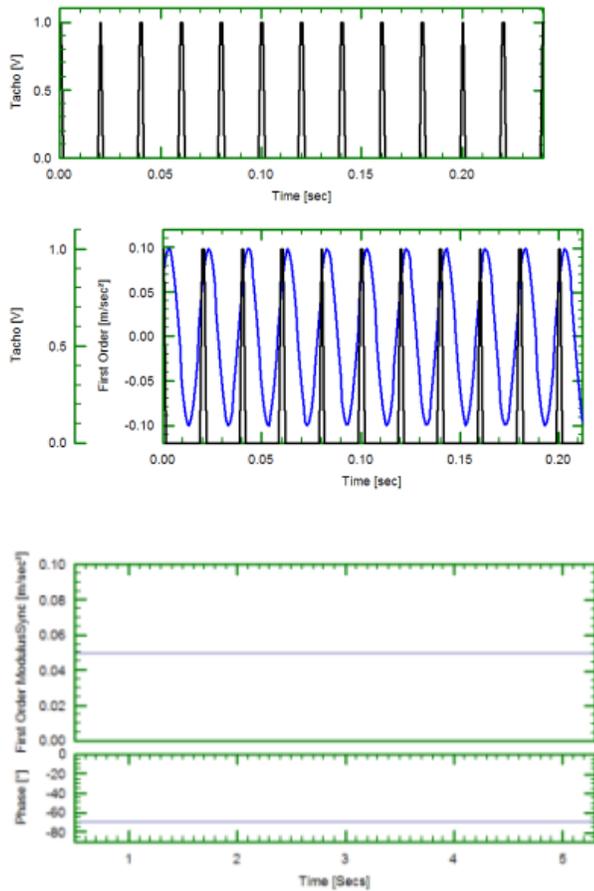
An example of this is the phase of an imbalance component in a rotor with reference to a fixed point on the rotor, such as a key way. To measure this phase, a trigger-pulse must be generated from a certain reference point on the shaft. This trigger can be generated by a tachometer or some type of optical or magnetic probe that senses a discontinuity on the rotor, and is sometimes called a "tach" pulse.



A zero degree phase delay at a frequency can be depicted as a series of pulses overlaid with a sine wave where the pulse edge is exactly located in peak position of the sine wave.



In Figure X a section of the tacho signal is shown on its own and then overlaid on the vibration signal. The tacho signal in this example crosses the vibration signal at exactly the same point on each cycle. If the phase of the vibration signal were to change then its position relative to the tacho pulse would also change. Extracting the first order modulus and phase, as before, gives the curves shown in Figure Y. The phase is now constant near  $-60^\circ$  as it should be for such a signal. Because the rotating period of the signal is about 20ms,  $-60^\circ$  corresponds to a  $20 \times 60 / 360 = 3.3$ ms delay.



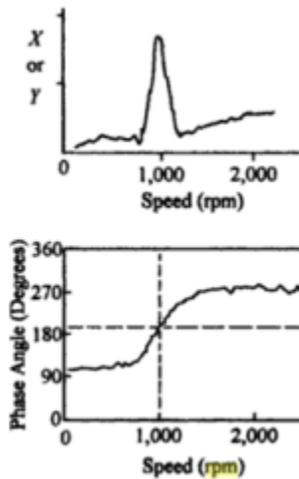
The phase measurement at higher orders will have the similar physical interpretation although they are difficult to comprehend intuitively.

It must be noted that the order tracks with phase, or Complex Order Tracks by name, are not regular complex signals as frequency response or cross spectrum. They are really auto-spectra with assigned phase. These synthesized signals can certainly be viewed as a complex signal using tools including Bode Plot, polar and orbit diagram. However the user must keep in mind that the magnitude and phase of a complex order track are calculated separately.

In the following sections, we will present how the order tracks with phase can be presented graphically with the Bode, polar and orbit plot.

### **Bode Plot**

The term Bode Plot is borrowed from the field of control theory, referring to a plot of magnitude and phase angle between the input and output versus frequency of a control system. Many in the rotating machine vibration industry have adopted this term to describe the steady-state vibration response amplitude and phase angle versus rotational speed (RPM). It turns out that the Bode Plot is the best way to describe order tracks with phase. You typically use Bode plots for transient analysis in both start-up and coast-down conditions. A Bode plot can help to identify the resonance speed of a rotor or examine the rotor dynamics on an order basis. A typical Bode Plot for an order track is shown below:

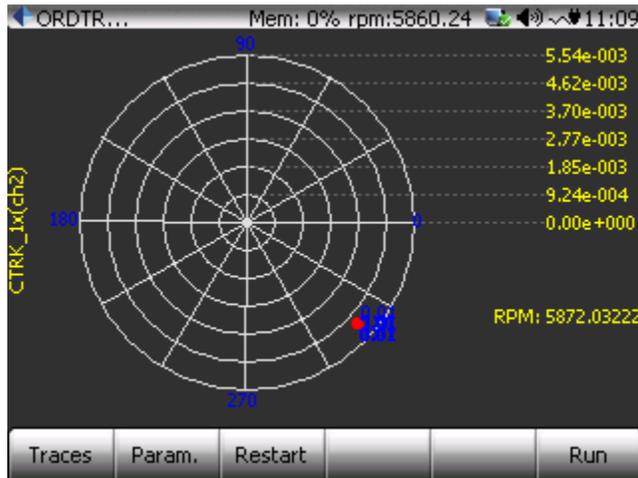


■ Figure 34 This picture must be replaced

In the CoCo system, after the order tracks are acquired together with their phase information then the Bode Plot can show one or multiple tracks.

### Polar Plot

The Polar Plot is another useful tool to view the order tracks for both amplitude and phase information. A polar plot draws the amplitude and phase on a polar coordinate. A typical polar plot is shown below.



■ Figure 35. Polar plot shows magnitude and phase on a polar axis.

In the polar plot, the dot shows the current order track value. The distance between the dot and the center indicates the magnitude of the order track while the angle corresponds to the phase measurement. The polar plot only shows the instantaneous measurement. It cannot keep the history versus RPM.

The Polar plot is often used to visually indicate the imbalance of the rotor. Polar plots and Bode plots often are combined to describe the rotating speed vector signal locus during speed changes. A Bode plot provides excellent change visibility with respect to speed, while the polar plot shows improved phase variation resolution.

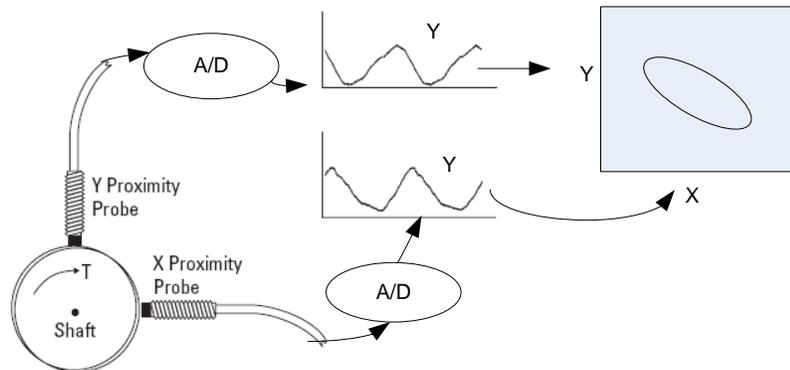
### Orbit Display

The Orbit display uses two data channels in the time domain. The signals from two channels are drawn on an X and Y plane to display the shaft position change versus angle of rotation. Orbit displays give a two dimensional visual picture of the motion of a rotating shaft.

A well balanced shaft with no movement in any direction and would produce a dot in the middle of the plot. The shaft movement can give an indication of the vibration source e.g. if there is a lot of up/down movement it may be that the machine feet are not bolted down tightly enough.

To create an orbit plot you need to take a dual channel simultaneous measurement to capture data at the horizontal and vertical axes at the same time. The displacement or acceleration sensors must be placed 90° apart from each other.

Orbit display uses a pair of measurement in time domain. It does not need the technique of *order tracking*.



The orbit display is similar to the Polar display in that it only displays the instantaneous status at the current RPM. In theory, the orbit display does not need a tachometer or another reference signal because X and Y reference to each other.

### Summary

With the Bode, polar and orbit plots, the order tracks with phase can be presented visually. These are effective tools used for rotating machine analysis. Bode plot is mostly used in the Run Up/Coast Down process. Polar plot and orbit, which only show the instantaneous status of an order at current RPM, are adequate for applications at steady or quasi-steady rotating speed such as dynamic balancing.

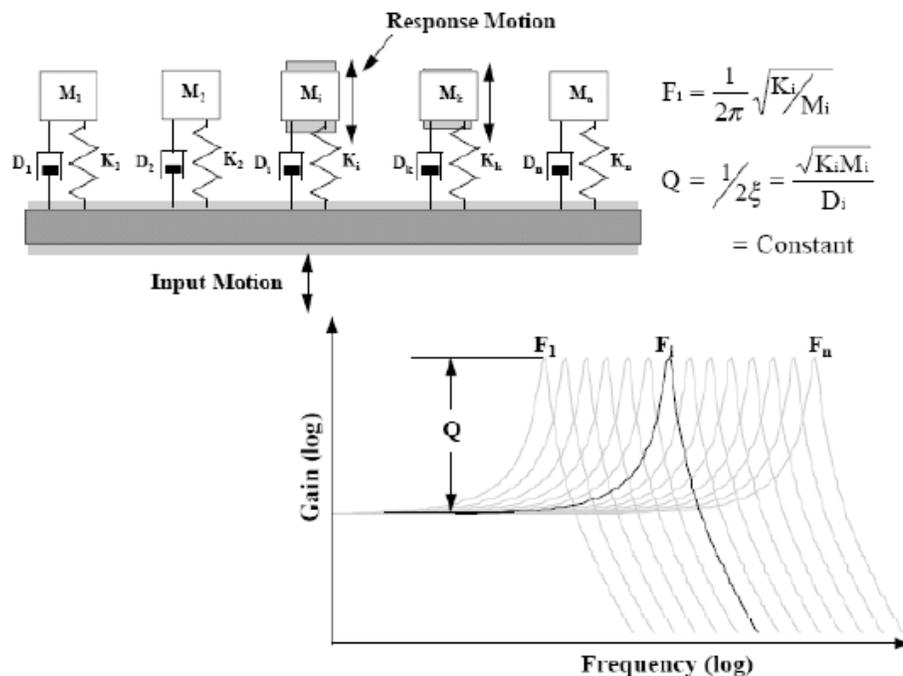
## 6. SHOCK RESPONSE SPECTRUM ANALYSIS

A **Shock** Response Spectrum (SRS) is a graphical representation of an arbitrary transient acceleration input, such as shock in terms of how a Single Degree Of Freedom (SDOF) system (like a mass on a spring) responds to that input. Actually, it shows the peak acceleration response of an infinite number of SDOFs, each of which have different natural frequencies. Acceleration response amplitude is represented on the vertical axis, and natural frequency of any given SDOF is shown on the horizontal axis.

A SRS is generated from a shock waveform using the following process:

1. Pick a damping ratio for the SRS
2. Assume a hypothetical Single Degree of Freedom System (SDOF), with a damped natural frequency of  $x$  Hz
3. Calculate (by time base simulation or something more subtle) the maximum instantaneous absolute acceleration experienced by the mass element of your SDOFs at any time during (or after) exposure to the shock in question. Plot this in g's (g's are standard, but pick any unit of acceleration you want) against the frequency ( $x$ ) of the hypothetical system.
4. Repeat steps 2 and 3 for other values of  $X$ , for example, logarithmically up to  $1000x$ .

The resulting plot of peak acceleration vs. test system frequency is called a Shock Response Spectrum, or SRS. This process can be depicted in the following picture:



■ Figure 36. Illustration of multi-degree of freedom system model used to compute SRS.

A SDOF mechanical system consists of the following components:

- Mass, whose value is represented with the variable, M
- Spring, whose stiffness is represented with the variable, K
- Damper, whose damping coefficient is represented with the variable C.

The resonance frequency,  $F_i$ , and the critical damping factor,  $\zeta$ , characterize a SDOF system, where:

$$F_i = \frac{1}{2\pi} \sqrt{\frac{K}{M}}$$

$$\zeta = \frac{c}{2\sqrt{KM}}$$

For light damping ratio where  $\zeta$  is less than or equal to 0.05, the peak value of the frequency response occurs in the immediate vicinity of  $F_i$  and is given by the following equation, where Q is the quality factor:

$$Q = \frac{1}{2\zeta}$$

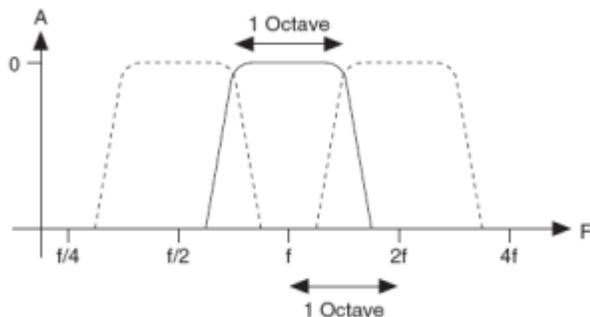
Any transient waveform can be presented as an SRS, but the relationship is not unique; many different transient waveforms can produce the same SRS (something one can take advantage of through a process called "Shock Synthesis"). The SRS does not contain all the information about the transient waveform from which it was created because it only tracks the peak instantaneous accelerations.

Different damping ratios produce different SRSs for the same shock waveform. Zero damping will produce a maximum response. Very high damping produces a very flat SRS. The level of damping is demonstrated by the "quality factor", Q which can also be thought of transmissibility in sinusoidal vibration case. A damping ratio of 5% results in a Q of 10. An SRS plot is incomplete if it doesn't specify the assumed Q or damping ratio value.

## Frequency Spacing of SRS Bins

Usually the SRS spectrum consists of multiple bins distributed evenly in the logarithmic frequency scale. The frequency distribution can be defined by two numbers: a reference frequency and the fractional octave number, such as 1/1, 1/3 or 1/6.

An octave is a doubling of frequency. For example, frequencies of 250 Hz and 500 Hz are one octave apart, as are frequencies of 1 kHz and 2 kHz.



■ Figure 37. Full octave filter shape.

The proportional bandwidth property will divide the frequency information uniformly over a log scale. It is very useful in analyzing a variety of natural systems. For example, the human response to noise and vibration is very non-linear and many mechanical systems have a behavior that is best characterized by proportional bandwidth analysis.

To gain finer frequency resolution, the frequency range can be divided into proportional bandwidths that are a fraction of an octave. For example, with 1/3 octave spacing, there are 3 SDOF filters per octave. In general, for 1/N fraction octave, there are N band pass filters per octave such that:

$$f_{c j+1} = f_{c j} * 2^{1/N}$$

where 1/N is called the fractional octave number.  $f_r$ , the **reference frequency**, is simply any of the frequencies  $f_{c j}$ . All other center frequencies of SDOF filters reference to this frequency. When the reference frequency and the fractional octave number are fixed, the frequency distribution over the whole frequency range is determined.

## SRS Measurement Quantities

Measurement quantities available to the CoCo SRS test are: time stream of each channel (raw data), block captured time signals and three SRS of each channel.

**Time streams:** this is the same as any other applications on the CoCo. Time streams are always available for viewing and recording. It is a very useful tool to observe whether the input signals are in the valid range. The recorded sine wave can be used for further post-processing. In CoCo, the time streams are often denoted as ch1, ch2 etc.

**Block time signals:** These are the block captured signals that are used for SRS analysis. Acquisition Mode will control how the block time signals are acquired.

**SRS:** Shock Response Spectra will be calculated for each block time signals. The engineering unit of the spectrum is determined by the sensor used by the input channel. In CoCo, the spectra are often denoted as three types: Maximum Positive spectrum; Maximum Positive spectrum and Maximum-Maximum spectrum.

**Maximum Positive Spectrum:** This is the largest positive response due to the transient input, without reference to the duration of the input.

**Maximum Negative Spectrum:** This is the largest negative response due to the transient input, without reference to the duration of the input.

**Maximax Spectrum:** this is the envelope of the absolute values of the positive and negative spectra. It is the most often used SRS type. The log-log Maximax is the universally accepted format for SRS presentation.

Other common SRS measures include the so called Primary SRS, Residue SRS and Composite SRS. The CoCo only calculates the Composite SRS.

## 7. AUTOMATED TEST AND LIMIT CHECK

Automated limit testing allows engineers and technicians to set up a pass/fail measurement on Any measured signal. This feature automates the process of determining whether an acquired signal meets, or is within a given set of criteria.

A limit test typically consists of comparing a waveform to upper and lower boundaries which the measured waveform must not cross. These boundaries are typically defined by the user to specify a tolerance band around a waveform. If any part of the waveform falls outside the limit, the software returns a failure message and the location of the failure on the waveform.

### Application Examples

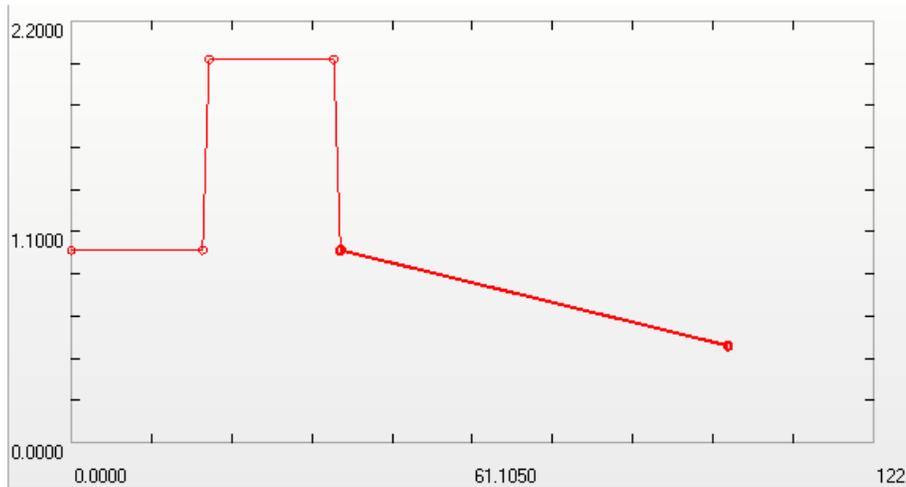
A common example for automated testing is related to structural testing. When excited, a structure will resonate at its natural frequencies. Structures can be excited through impact or by other means. Structural defects can result in a shift in resonant peaks. Therefore, in structural tests, frequency 'alarms' are used to monitor the frequency response in areas of spectral interest.

Another example of automated testing is related to rotating machinery. Rotating or moving assemblies produce vibration and noise patterns that can be examined to identify the fingerprint of 'good quality.' Product defects will cause additional spectral peaks, or changes in peak levels. Therefore, in 'self-excited' product testing, 'level alarms', which can be set to trigger on peak or RMS values, are placed around the areas of spectral interest. Unwanted signals, from background noise, are therefore ignored.

### Testing Limit Signals and Testing Schedule

Automated testing can be performed on a wide variety of signals including a time domain capture, an auto-power spectrum, an octave spectrum, an order track signal or a frequency response. The CoCo instrument compares the limits to the live measured signal in real-time, after every single frame of measurement. If the limits are exceeded, the CoCo takes the appropriate actions based on the user setup.

An upper and or lower limit can be applied to a signal to be tested. Limit signals are constructed by defining breakpoints. A breakpoint is controlled by a pair of X/Y values. Figure 38 shows a typical automated test limit signal with 4 breakpoints.



■ Figure 38. Typical automated test limit signal with 4 breakpoints.

To automatically control the limit checking test, a **testing schedule** is developed for CoCo. The testing schedule defines the various operations to automate the process. For example the testing schedule can tell the instrument when the limit checking will be turned on, when it will be turned off and for how long the test will be conducted.

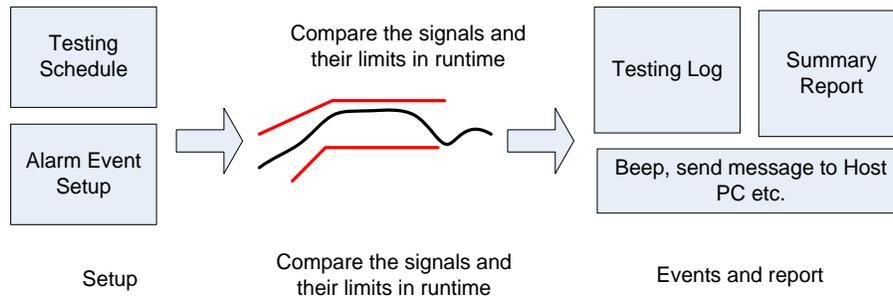
To record the events of the test a Testing Log and a summary report are needed. The Testing Log records the important events, including whether the limits are exceeded, in chronicle order. The Summary Report provides the status of limiting check since the last time when the test was started.

When the limit signal is exceeded then a user defined limit alarm event will be triggered. This can include an audible beep, Save Signals, send messages and so on.

To summarize, an automated limit checking test requires the following building blocks:

- At least one test signal
- At least one limit signal applied to the measured signals
- A testing schedule
- A testing log and summary report
- A setup for the limit alarm events

Figure 39 illustrates the automated testing process



■ Figure 39. Illustration of automatic testing process.

**Test Signals:** Any block signal can be used for testing. Typically the test signals are time captured blocks, auto power spectra, frequency response, octave spectra or order tracks. Time streams are not used for limiting test. **Limit signals:** Limit signals including upper and/or lower limits are defined in the CSA Editor. Limit signals are applied to testing signals. Up to a maximum of 64 segments can be defined for each limit signal. The maximum number of limit signals is 64.

**Testing Schedule:** The testing schedule automatically controls the test using an event driven process. Multiple testing schedules can be developed and one is executed at a time. Testing schedule event entries include: Loop/End-Loop, Set Sampling Rate, Set All Input Mode, Run Duration, Hold, Limit Check on, Limit Check off, Start Recording, Stop Recording, Save Signals, Turn Signal Source On and Turn Signal Source Off.

**Testing Log:** A log file is automatically created for each run of the schedule to record major events.

**Limit Check Alarm Event Setup:** Events include an audible beep from the CoCo, CoCo screen flashing, entry into the Testing Log, send message to host PC via EDM software and Save Signals.

When a limit is exceeded, the predefined events are triggered. For example, the CoCo may beep, flash the screen, save the signals to the storage device, or send the message to the host PC.

## Networked CoCo used for Automated Test

CoCo has an Ethernet network interface that provides a unique advantage that multiple CoCo units can be connected remotely using an Ethernet network. This is particularly useful for vibration monitoring or production test that requires long distance access.



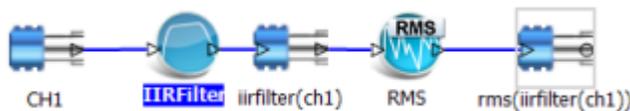
■ Figure 40. Remote operation for multiple CoCo units with automated testing.

When multiple CoCo units are managed through the EDM software, the host computer can record or react to the alarm events from each remote CoCo unit.

## 8. REAL TIME DIGITAL FILTERS

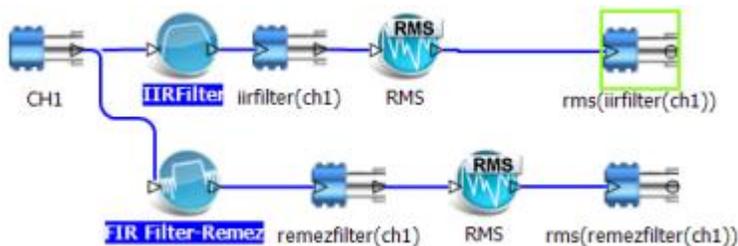
Real Time Digital Filters is a powerful analysis tool that can be used to filter a measured signal in real time and then apply the FFT and time based analysis built into the CoCo. You can precisely define the filter characteristics to meet your specific application. The filter definition is performed in the EDM software and the filter is included in the CSA file that is downloaded to the CoCo. This capability in a small portable unit makes the CoCo a very powerful analysis tool.

For example, a user might want to look at the energy distribution over time, for a specific band of frequencies instead of the entire frequency spectrum from zero to the maximum sampling rate. This can be done by creating a band-pass filter then applying an RMS estimator to the output of the filter. Figure 41 shows the graphical representation of this process which is used to define the real time filter in the EDM software. The icon on the left, CH1 represents the native measured time stream. It is connected to an IIR Filter which computes a signal named `iirfilter(ch1)` which is connected to an RMS estimator. The output of the RMS estimator is a signal named `rms(iirfilter(ch1))`. The EDM software will be discussed in more detail later.



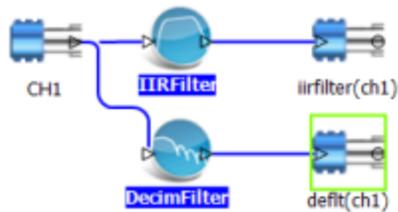
■ Figure 41. Example real time digital filter application.

Another example is that the user might want to look at the frequency energy over 100Hz to 200Hz and 1000Hz to 2000Hz separately. This can be done by deriving two output streams from the native channel 1, then applying the band-pass filter to each path as shown in Figure 42.



■ Figure 42. Digital Real Time Filter example with two output streams.

In another example, a user might want to look at the very fast time characteristics of a channel at high frequency, and the same channel at a very low sampling rate. This can be done by applying a decimation filter to the native time stream as shown in Figure 43. The native channel time stream is split into two streams so the signal from the same channel is recorded at both high and lower sampling rates.



■ Figure 43. Example computing high and low sampling rate with a decimation filter.

The Real Time Digital Filters option includes three types of digital filters: FIR, IIR and decimation filters. For FIR and IIR filter, you can specify low-pass, high-pass, band-pass or band-stop types with several different methods. This chapter first explains the theory about the filter design, and then introduces the operations within the CSA Editor and CoCo hardware.

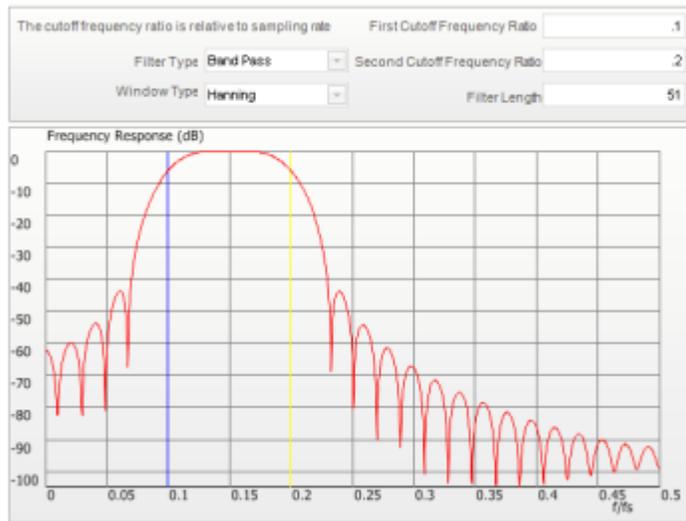
Real Time Filter implementation can be divided into two steps that include the filter definition on the EDM software, and secondly download and run the CSA on the CoCo hardware.

You design a filter based on certain criteria such as cut-off frequency, pass band ripple, attenuation level and so on. The EDM software walks you through this process. The outcome of this design process is simply a number of filter coefficients that represent the filter which are included in a CSA. The software will upload the CSA including the filter coefficients to the CoCo hardware.

After filter is defined and the coefficients downloaded to the CoCo hardware you can run the CSA. When the CoCo is running, the filter coefficients created in the filter design process will be used. The time streams will pass through the filters and generate new time stream signals.

The goal of filter design is to calculate a series of filter coefficients, also known as “taps” based on the user specified criteria. The criteria are often described by following variables:

- **Number of filter coefficients:** this is also known as the order of the filter. The filter order defines how many coefficients are required to define the filter. A lower order filter consists of a fewer number of coefficients. A low order filter responds relatively faster than a higher order filter, that is there is less of a time lag in the output of the filter.
- **Cutoff frequencies:** For low pass or high pass filters, only one cutoff frequency is needed. Band pass or band stop filters require two cutoff frequencies to fully define the filter shape. Figure 44 shows a typical band pass filter design with the two cutoff frequencies set to approximately 0.1 and 0.2 Hz as indicated by the blue and yellow vertical lines.
- **Attenuation of stop band in dB:** This defines how much of the input signal is cut out of the output at the rejected frequencies. In theory the higher the attenuation the better. In Figure 44 the stop band attenuation is > 40 dB as seen from the highest side lobe just below 0.25 Hz.
- **Pass band ripple:** Ripple is an unavoidable characteristic of a digital filter. It refers to the fluctuation in the filter shape at transition frequencies. If a very flat filter is required then it can be specified by choosing a very low ripple. In Figure 44 ripple is seen in the stop band and no ripple is evident in the pass band. Ideally the pass band should be very flat and some ripple is tolerable in the stop band.
- **Width of transition bands:** This refers to the filter shape between a band pass and a band stop region. Ideally this transition band should be very small. However, a very narrow transitional band requires a higher order filter which affects the filter response time and can also affect ripple. In Figure 44 the transition bands are between 0.05 to 0.1 and 0.2 to 0.25.



■ Figure 44. Filter design shows cutoff frequencies, ripple, band stop attenuation.

In most cases filter design includes making tradeoffs between minimizing the filter order, ripple, transition band width and response time. Not all can be satisfied at the same time. Filter design can be an iterative process and experience is helpful.

## FIR Real Time Digital Filters

Finite Impulse Response (FIR) filters have the distinctive trait that their impulse response lasts for a finite duration of time as opposed to, an Infinite Impulse Response (IIR) filters whose impulse response is infinite in duration. This trait is due to the fact that there are no feedback paths in the FIR filter. FIR filters offer several advantages over IIR filters:

- Completely constant group delay throughout the frequency spectrum. Group delay refers to the time delay between when a signal goes into the filter and when it comes out. Constant group delay means that an input signal will come out of the filter with all parts delayed the same amount with no distortion.
- Complete stability at all frequencies regardless of the size of the filter.

FIR filters also have some disadvantages as well:

- The frequency response is not as easily defined as it is with IIR filters
- The number of coefficients required to meet a frequency specification may be far larger than that required for IIR filters.

CoCo allows up to 128 taps (orders) for the real-time FIR filter.

A digital filter can be understood by considering the difference equation which defines how the input signal is related to the output signal as

$$y[n] = b_0x[n] + b_1x[n - 1] + \dots + b_Nx[n - N]$$

where  $x[n]$  is the current input signal sample,  $x[n-1]$  is the previous signal sample and  $x[n-N]$  is the last sample in the series. The series multiplies the most recent  $N+1$  samples with associated the  $N+1$  filter coefficients.  $y[n]$  is the current output signal and  $b_i$  are the filter coefficients. The number  $N$  is known as the *filter order*, an  $N^{\text{th}}$ -order filter has  $(N + 1)$  terms on the right-hand side and  $N+1$  filter coefficients also referred to as “*taps*”.

This equation illustrates why a higher order filter has a slower response time. It takes more samples and therefore more time for an event to work its way through the series until the output is no longer affected by the event as compared to a lower order filter with fewer coefficients.

The previous equation can also be expressed as a convolution of the filter coefficients and the input signal.

$$y[n] = \sum_{i=0}^N b_i x[n - i]$$

The Impulse Response of the filter shows how the historical data affect the current filtered value. The longer the impulse response, the farther the old data will affect the current filtered value. To find the impulse response we set

$$x[n] = \delta[n]$$

where  $\delta[n]$  is the Kronecker delta impulse. The equation below shows that the impulse response for an FIR filter is simply the set of coefficients  $b_n$ , as follows

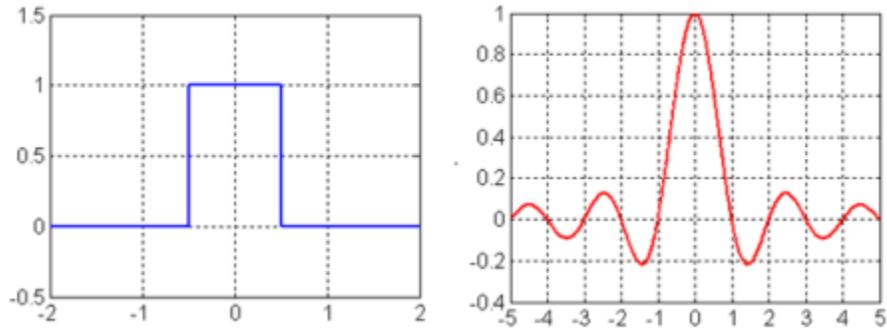
$$h[n] = \sum_{i=0}^N b_i \delta[n - i] = b_n \quad \text{for } n = 0 \text{ to } N$$

FIR filters are clearly stable, since the output is a sum of a finite number of finite multiples of the input values, so can be no greater than  $\sum_{n=0}^N |b_n|$  times the largest value appearing in the input.

### Data Windows FIR Filter Design

In the academic world, hundreds of methods are available to design an FIR filter to meet various criteria. The EDM includes the most popular filter design methods: Data Window and Remez. Both methods are discussed below.

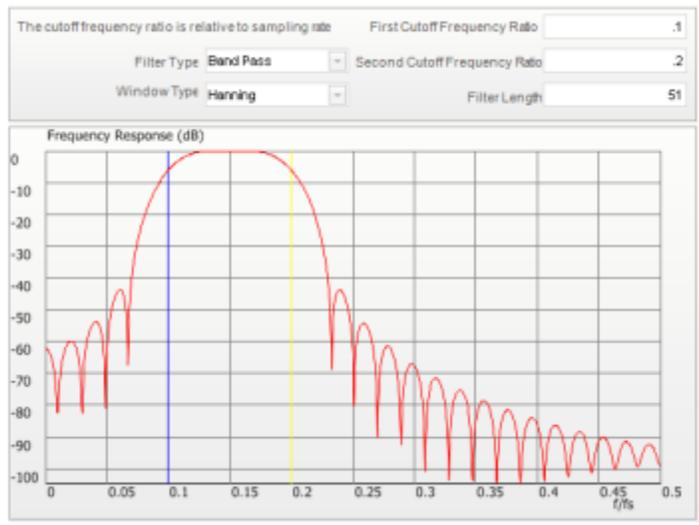
The Data Window FIR Filter Design method is the easiest to understand. The name "Window" comes from the fact that these filters are created by scaling a *sinc* ( $\text{SIN}(X)/X$ ) function with a window such as a Hanning, Flat Top, etc. to produce the desired frequency effect.



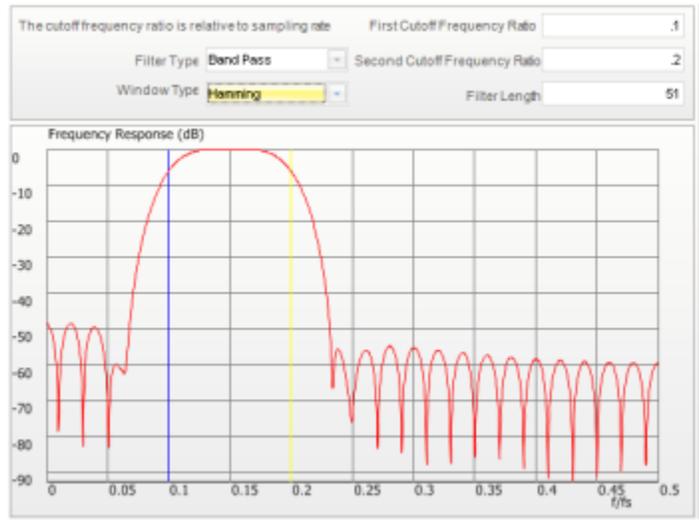
■ Figure 45 Sinc function is the Fourier transform of a square shape.

A data window FIR filter is generated by starting with an ideal “brick-wall” shaped filter, that is a filter with vertical edges or zero transition band width as shown on the left in Figure 45. The brick-wall filter is specified by the cutoff frequencies and has a band pass amplitude of 1 and a stop band amplitude of zero. The problem with the ideal brick-wall filter is that the time response oscillates forever and it requires an infinite number of filter coefficients. This ideal filter can be modified by applying a data window to force the time response to decay in a finite time. Of course this degrades the shape of the ideal brick-wall filter performance. It introduces ripple, increases the transition band width and increases the stop band attenuation. However it allows the filter to be defined by a finite number of filter coefficients. The filter performance can be modified by using different data windowing functions and making the tradeoff between filter order and response time. The user must choose these settings during the filter design.

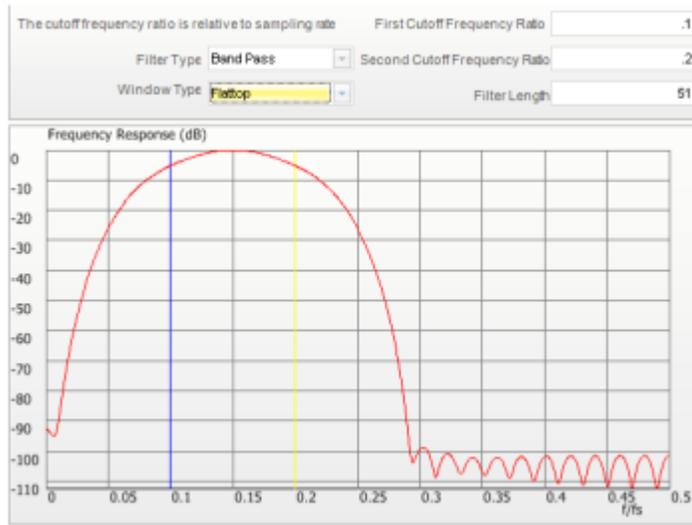
Figure 46 through Figure 51 show a comparison of different data window choices for the same filter settings. In all cases the low and high cutoff frequencies are 0.1 and 0.2 relative to the sampling frequency. The number of filter taps is 51.



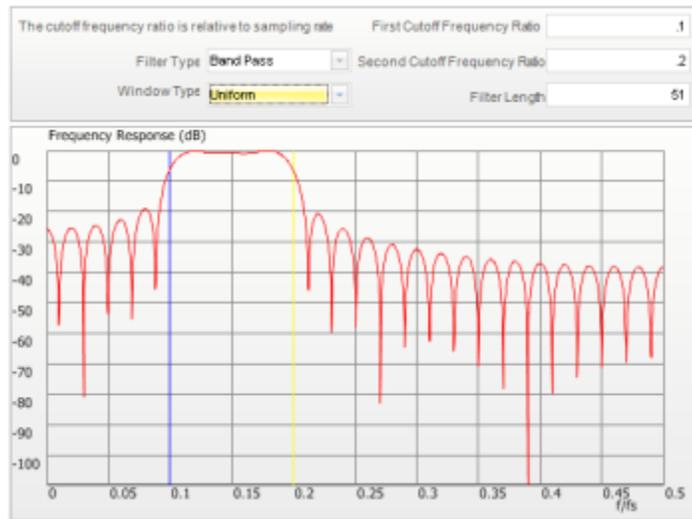
■ Figure 46 Hanning window method.



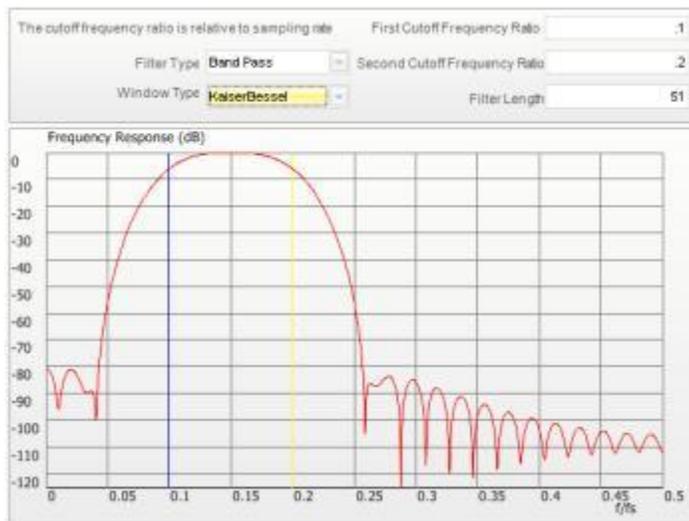
■ Figure 47 Hamming window method.



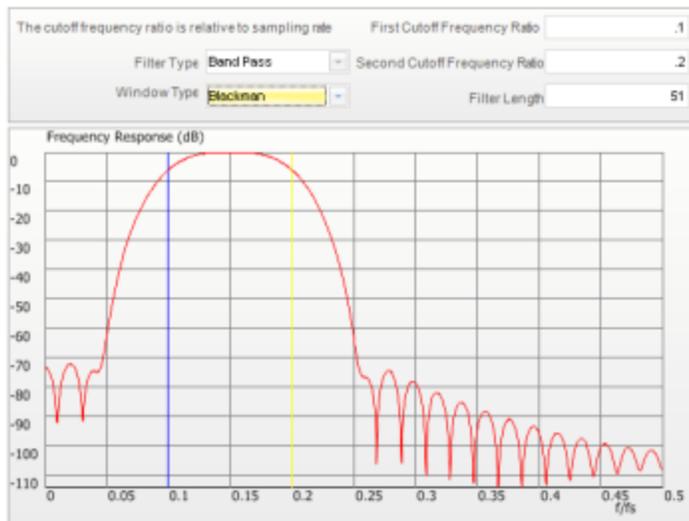
■ Figure 48 Flattop window method.



■ Figure 49 Uniform window method.



■ Figure 50 Kaiser Bessel window method.



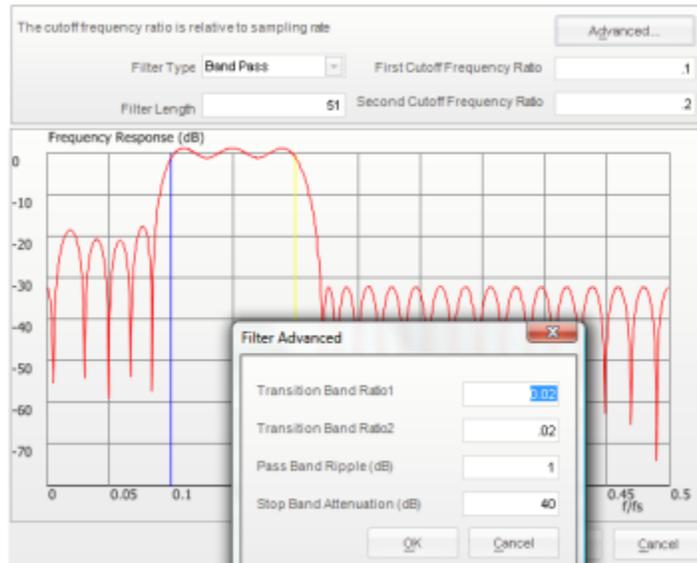
■ Figure 51 Blackman window method.

As shown in the pictures, different window methods produce different filter performance, i.e., different attenuation of the main lobe and side lobes. The best data window choice depends on your specific application. Refer to the Basic Spectral Analysis section for a comparison of windowing functions.

### Remez Filter Design

The Remez Filter is a different method for designing an FIR filter. It is more computationally intensive than the data window method. A Remez filter is generated with iterative error-reducing algorithms designed to reduce the pass band error. In addition to allowing stop band ratio and frequency definition, the Remez filter allows the "Ripple Ratio" to be defined as a user specified parameter.

Figure 52 shows an example of a filter design using the Remez method in the EDM software. The low and high cutoff frequencies are 0.1 and 0.2 relative to the sampling frequency. The number of filter taps is 51.



■ Figure 52. Remez FIR Filter design dialog.

The software is intelligent enough to automatically calculate the total FIR filter length based on these criteria. For example if the user asks for very high attention, very small ripple or very sharp transition band, the filter length will go very high. The user must make tradeoffs between these parameters so that appropriate filter length can be generated and used.

## IIR Real Time Digital Filters

**Infinite impulse response (IIR)** filters have the trait that their impulse response decays very slowly but theoretically lasts forever. This is due to the fact that the filter input includes the measured signal and also the filter output creating a feedback path which results in the infinite impulse duration. This is in contrast to finite impulse response filters (FIR) which have fixed-duration impulse responses.

The design procedures for IIR filters is somewhat more complicated than FIR filter design because there is no direct design method like the data window method for FIR filters. Instead IIR filters are typically designed by starting with an ideal analog filter in terms of the frequency response characteristics such as the Chebyshev, Butterworth, or Bessel filter. Then the analog filter is converted into a digital filter using a method known as the Bilinear transformation or impulse invariance method.

An IIR digital filter can be understood by considering the difference equation which defines how the input signal is related to the output signal as

$$y[n] = b_0x[n] + b_1x[n - 1] + \dots + b_px[n - P] - a_1y[n - 1] - \dots - a_qy[n - Q]$$

where P is the feed-forward filter order,  $b_i$  are the feed-forward filter coefficients, Q is the feedback filter order,  $a_i$  are the feedback filter coefficients,  $x[n]$  is the input signal and  $y[n]$  is the output signal.

The previous equation can also be expressed as a convolution of the filter coefficients and the input signal.

$$y[n] = \sum_{i=0}^P b_i x[n-i] - \sum_{j=0}^Q a_j y[n-j]$$

which, when rearranged, becomes:

$$\sum_{j=0}^Q a_j y[n-j] = \sum_{i=0}^P b_i x[n-i] \quad \text{if we let } a_0 = 1$$

To find the transfer function of the filter, we first take the Z-transform of each side of the above equation, where we use the time-shift property to obtain:

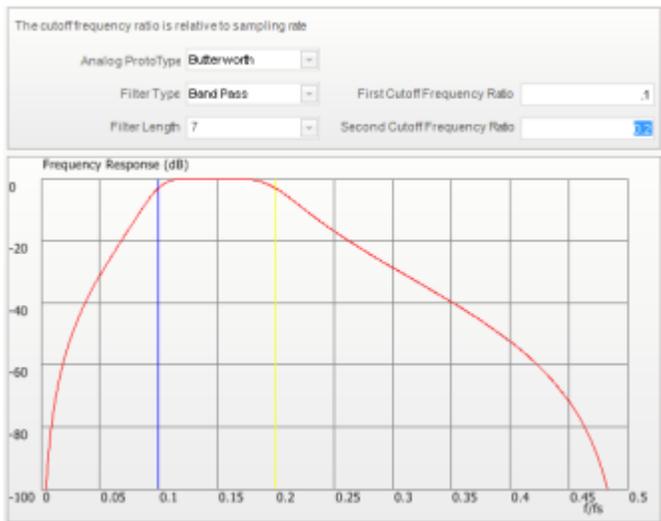
$$\sum_{j=0}^Q a_j z^{-j} Y(z) = \sum_{i=0}^P b_i z^{-i} X(z)$$

We define the transfer function to be:

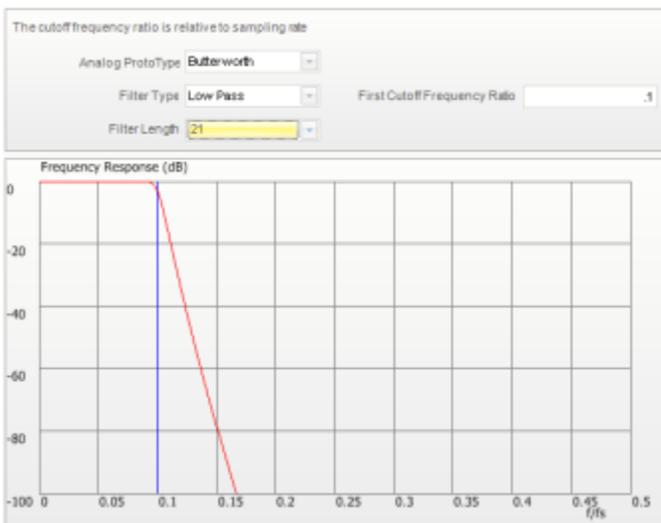
$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{i=0}^P b_i z^{-i}}{\sum_{j=0}^Q a_j z^{-j}}$$

The transfer function gives the frequency response that relates the input to the output magnitude and phase relationship.

Various analog filter types can be used as the basis for the IIR filter. The Butterworth Filter is the filter type that results in the flattest pass band and contains a moderate group delay. Below are examples of Butterworth low pass and band pass filters.

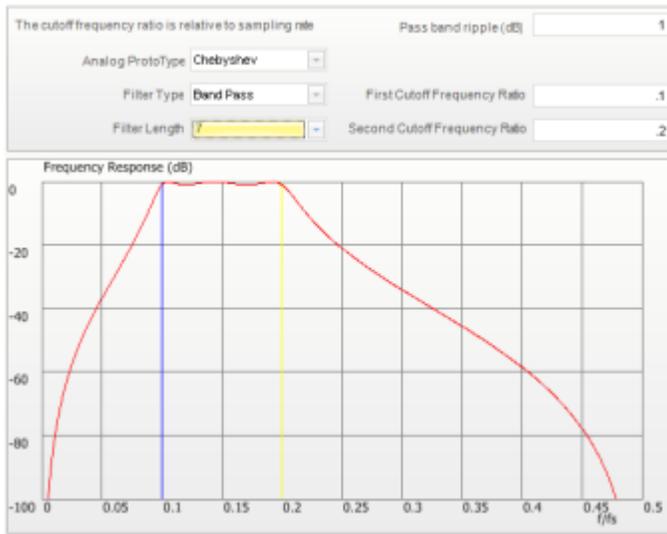


■ Figure 53. Butterworth band pass filter.

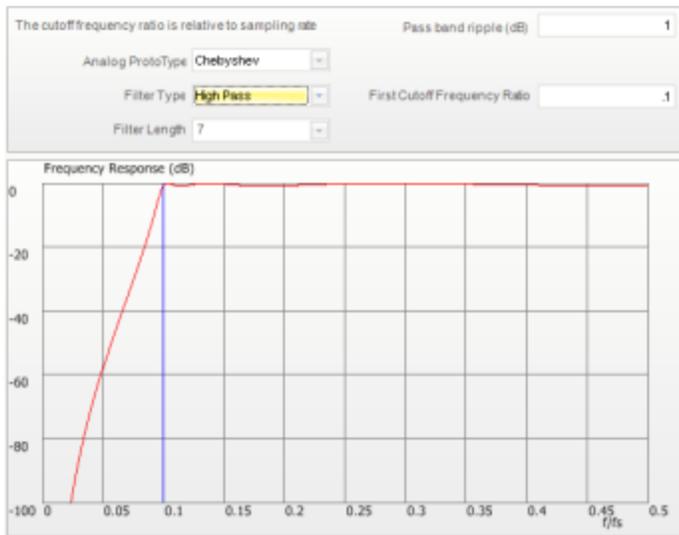


■ Figure 54. Butterworth low pass filter.

The Chebyshev Type I Filter results in the sharpest pass band cut off and contains the largest group delay. The most notable feature of this filter is the significant ripple in the pass band magnitude. A standard Chebyshev Type I Filter's pass band attenuation is defined to be the same value as the pass band ripple amplitude. Below are examples of Chebyshev Type I band pass and high pass filters.

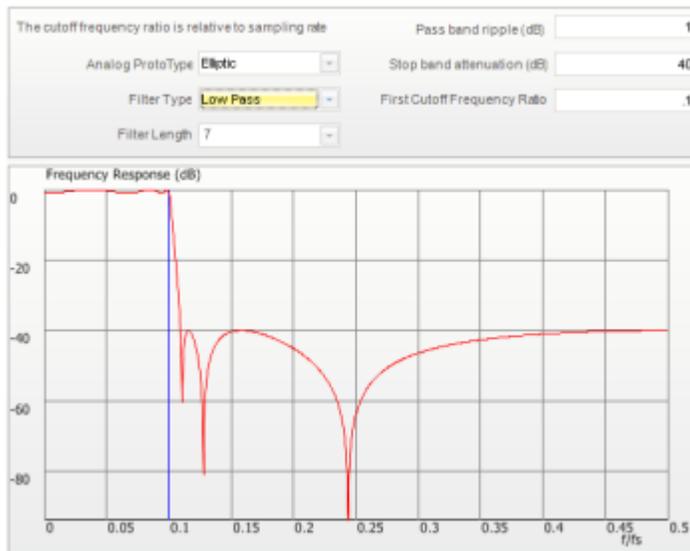


■ Figure 55. Chebyshev type I band pass filter.

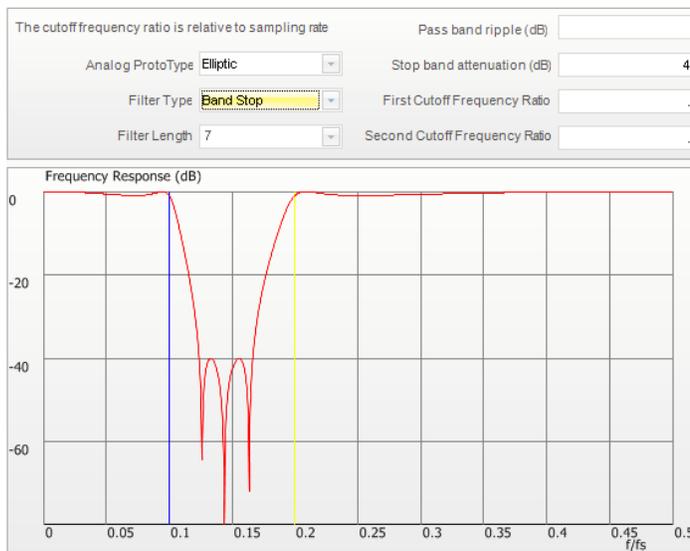


■ Figure 56. Chebyshev type 1 high pass filter.

The Elliptic Filter contains a Chebyshev Type I style equi-ripple pass band, an equipped stop band, a sharp cutoff, high group delay, and the greatest possible stop band attenuation. Below are examples of 7th order Elliptic low pass, band stop filters.



■ Figure 57. Elliptical low pass filter.



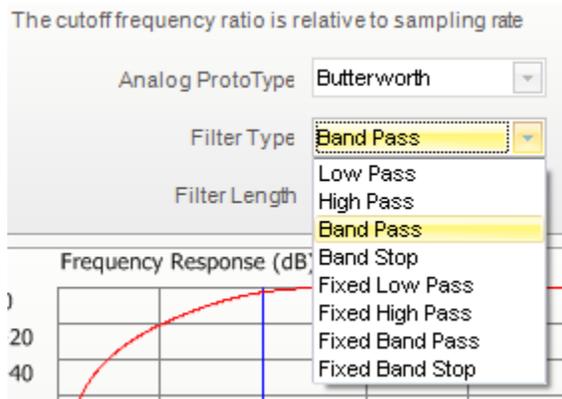
■ Figure 58. Elliptical band stop filter.

## Filter Design Using Fixed instead of Relative Frequency

Filter design can be accomplished using either fixed or relative frequency methods. In the relative frequency method the cutoff frequencies are defined relative to the maximum sampling rate. For example if the sampling rate is 1000 Hz and a low pass filter is defined with a cutoff frequency of 0.5 with the relative frequency method then the cutoff frequency is 500 Hz. Note that if the sampling rate is changed to 500 Hz and the cutoff frequency is no changed from 0.5, then the cutoff frequency will change to 250 Hz. The relative frequency method is the preferable method because the filter performance such as ripple or transition band width will not change when the sampling rate is changed.

The alternative is the fixed frequency method where the cutoff frequency is defined by a fixed frequency. With this method the cutoff frequencies do not need to be changed when the sampling rate is changed. While this method is more user friendly than the relative frequency method, it is not the recommended method because the filter performance, such as ripple or transition band width can vary when the sampling rate is changed.

To change between fixed or relative frequencies, go to Filter Type menu and select a filter type with Fixed. The filter types without Fixed are relative by default. :

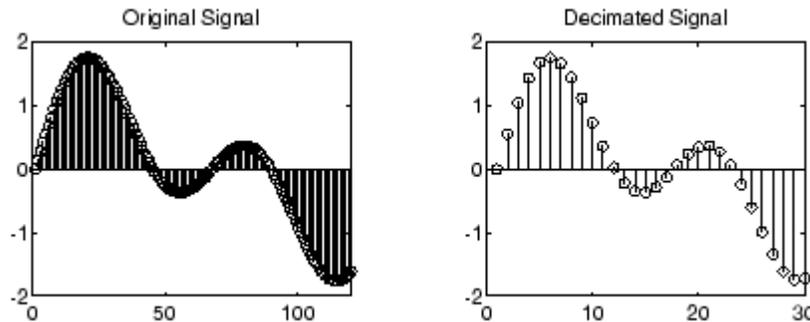


■ Figure 59. Fixed or relative frequency setting.

## Decimation Filters

The decimation filter is a special filter available on the CoCo. Decimation reduces the original sampling rate for a sequence to a lower rate. The decimation process filters the input data with a low-pass filter and then re-samples the resulting smoothed signal at a lower rate.

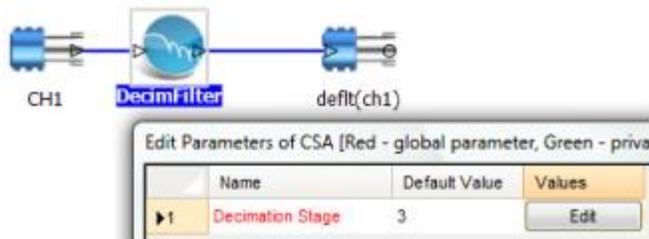
Figure 60. Illustration of a decimation filter. shows how a decimation filter reduce the number of sampled points from 150 to 30 while the signal shape which is dominated by the low frequency components is still retained.



■ Figure 60. Illustration of a decimation filter.

Low pass filtering is important in the decimation process to ensure no aliasing occurs. Aliasing refers to the effect of under sampling a high frequency signal and misrepresenting the high frequency behavior by a lower frequency. When aliasing occurs there is no way to distinguish the erroneous aliased signal from the actual signal. In the CoCo hardware the decimation filter uses a fixed proprietary low pass FIR filter with excellent ripple performance in the pass-band and very high attenuation in the stop band.

In the CoCo hardware, the decimation filter module contains multiple stages of decimation filters. In each stage the data is decimated by a factor of two. After N stages of decimation, the data will be reduced to its  $1/2^N$ . In the example below, since the decimation stage is set to 3, the data will be reduced to  $1/2^3 = 1/8$  of its input points after this decimation module.



■ Figure 61. Decimation filter in the EDM software.

The decimation filter is widely used to view, analyze and record low frequency signals. For example, a system may be used to acquire the vibration and pressure or temperature data simultaneously. While the signals are all sampled at the high data rate, the pressure channel and the temperate channel should be viewed and recorded at a much lower rate because these types of signals typically do not change dynamically (at a high frequency). In this case, we can simply apply decimation filters to the channels that measure and record pressure or temperature.

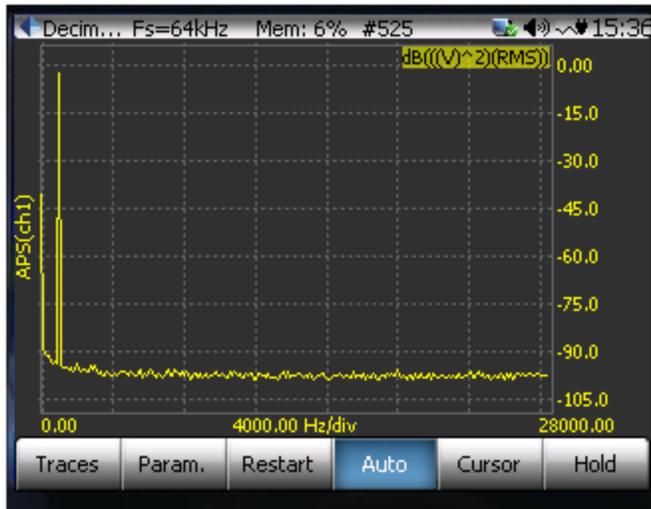
Integrating decimation filter with other filter techniques allows the user to analyze high frequency and lower frequency signals with different frequency resolution simultaneously. This capability is unique to the CoCo system.

For example, in a CSA file you can apply a 3-stage decimation filter to channel 1 as shown in Figure 62.



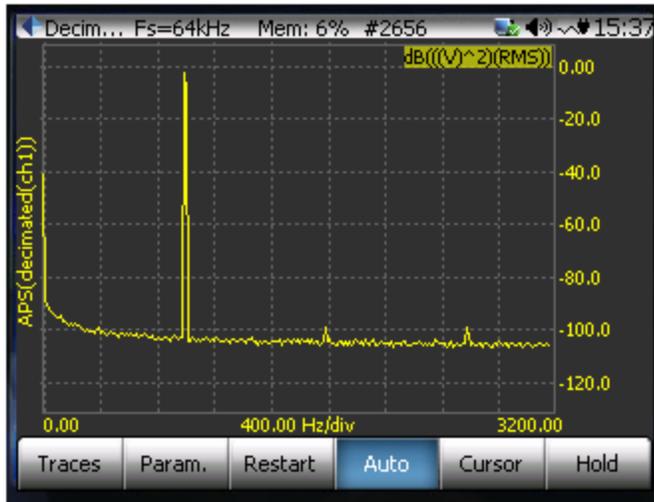
■ Figure 62. Decimation filter example.

If you connect a signal source with 1 kHz sine output to channel 1 and set the sampling rate to 64 kHz, you can see the broad spectrum up to 28 kHz as shown in Figure 63.



■ Figure 63 The auto spectrum of an 1kHz sine wave when sampled at 64kHz

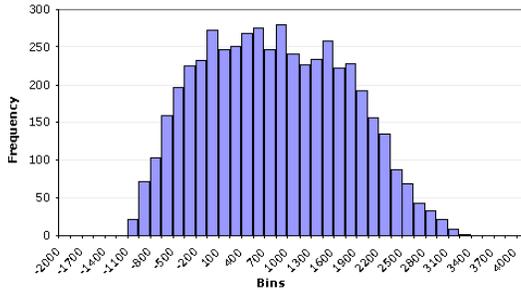
However you can also **simultaneously** show the spectrum of the decimated signal, *decimated(ch1)*, with 8 times frequency resolution as shown in Figure 64. Note that in this example the two spectra are of the exact same time signal not of two samples acquired at different times.



■ Figure 64 The same sine wave at 1kHz, after decimated 8:1, shows at different location on the spectrum

## 9. HISTOGRAM AND STATISTIC MEASURES

A histogram is a graphical display that shows the number (or frequency) of events that fall into each of several or many specified categories. Figure 65 shows a typical histogram.



■ Figure 65. Typical Histogram.

Mathematically, a histogram is a mapping  $m_i$  that counts the number of observations that fall into various disjoint categories (known as *bins*). Thus, if we let  $n$  be the total number of observations and  $k$  be the total number of bins, the total number of events can be found by adding the frequency in all the bins as

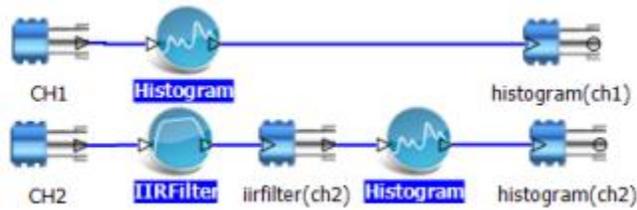
$$n = \sum_{i=1}^k m_i$$

### Cumulative Histogram

A cumulative histogram is a mapping that counts the cumulative number of observations in all of the bins up to the specified bin. That is, the cumulative histogram  $M_i$  of a histogram  $m_i$  is defined as:

$$M_i = \sum_{j=1}^i m_j$$

In CoCo implementation, the Histogram and Statistics function is a single CSA module that can be applied to *any* time stream. The output of the Histogram and Statistics module is a histogram signal and the associated statistics results. You can change the display format on CoCo.



■ Figure 66. Histogram example.

CoCo provides the following measurement parameters for a histogram: bin number for the bar chart and amplitude ranges. It provides the following display formats for the histogram graph: normalized-linear, normalized -logarithmic, un-normalized and cumulative. While the histogram is measured, it also provides the following statistics values: *mean*, *max*, *min*, *RMS*, *variance*, *skewness*, *crest factor* and *kurtosis*. The definitions of these statistics measures for N samples are:

$$\text{Mean} = \mu_x = \bar{x} = E(x) = \left( \sum_{i=1}^N x_i \right) / N$$

$$\text{Variance}(x) = \left( \sum_{i=1}^N (x_i - \bar{x})^2 \right) / N$$

$$\text{standard deviation } \sigma = \left[ \left( \sum_{i=1}^N (x_i - \bar{x})^2 \right) / N \right]^{1/2}$$

$$\text{rms}(x) = \left[ \left( \sum_{i=1}^N (x_i)^2 \right) / N \right]^{1/2}$$

The  $\text{rms}(x)$  is equal to the standard deviation when the mean is 0.

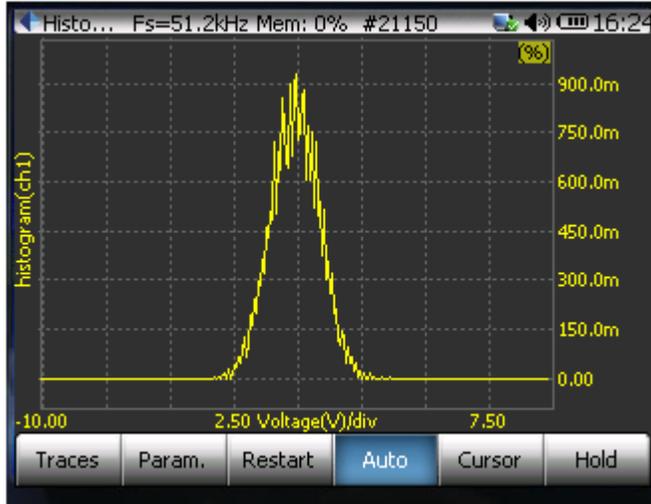
Skewness is a measure of the asymmetry of the data around the sample mean. If the skewness is negative, the data are spread out more to the left of the mean than to the right. If the skewness is positive, the data are spread out more to the right. The skewness of the normal distribution (or a perfectly symmetric distribution) is zero.

$$\text{Skewness}(x) = \left( \sum_{i=1}^N (x_i - \bar{x})^3 \right) / N\sigma^3$$

Kurtosis is a measure of how outlier-prone a distribution is. The kurtosis of the normal Gaussian distribution is 3. Distributions that are more outlier-prone than the normal distribution have kurtosis greater than 3; distributions that are less outlier-prone have kurtosis less than 3.

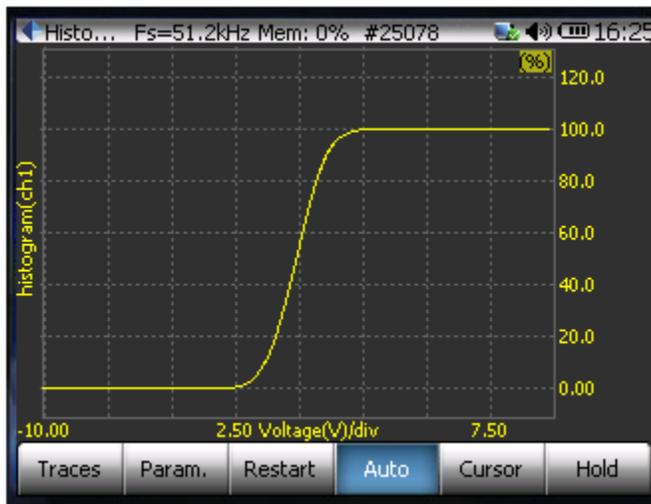
$$\text{Kurtosis}(x) = \left( \sum_{i=1}^N (x_i - \bar{x})^4 \right) / N\sigma^4$$

As an example, a histogram of a Gaussian random noise is displayed in Figure 67.



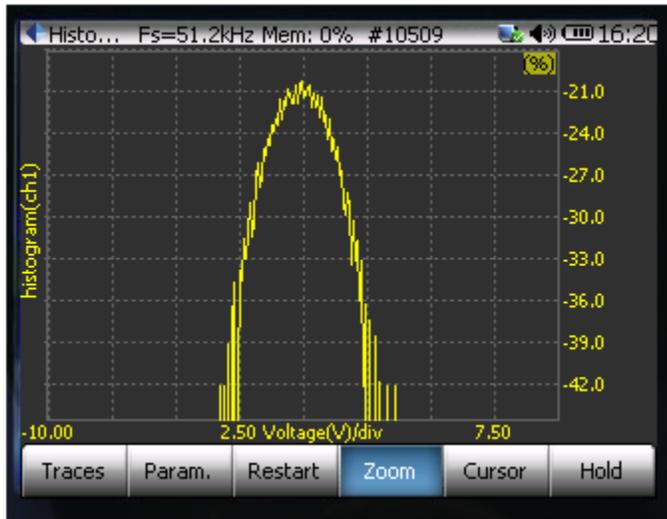
■ Figure 67. Histogram of a Gaussian distributed random signal.

Its cumulative histogram is displayed in Figure 68.



■ Figure 68. Cumulative Histogram of random Gaussian signal.

The display format can be changed to analyze the histogram. For example, a signal distribution with high Kurtosis can be observed using a logarithmic vertical scale with dB units, in the unit of dB, as shown in Figure 69. Here the low frequency outliers can more easily be seen.



■ Figure 69. Histogram with dB scale.

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